ABSTRACT
This paper combines the Taguchi-based response surface methodology (RSM) with a multi-objective hybrid quantum-behaved particle swarm optimization (MOHQPSO) to predict the optimal surface roughness of Al7075-T6 workpiece through a CNC turning machining. First, the Taguchi orthogonal array L_{27} (3^6) was applied to determine the crucial cutting parameters: feed rate, tool relief angle, and cutting depth. Subsequently, the RSM was used to construct the predictive models of surface roughness (\(R_a\), \(R_{\text{max}}\), and \(R_z\)). Finally, the MOHQPSO with mutation was used to determine the optimal roughness and cutting conditions. The results show that, compared with the non-optimization, Taguchi and classical multi-objective particle swarm optimization methods (MOPSO), the roughness \(R_a\) using MOHQPSO along the Pareto optimal solution are improved by 68.24, 59.31 and 33.80%, respectively. This reveals that the predictive models established can improve the machining quality in CNC turning of Al7075-T6.

Keywords: Taguchi method; RSM; MOHQPSO; CNC turning; surface roughness.

PRÉVISION OPTIMALE ET CONCEPTION DE LA RUGOSITÉ DE SURFACE POUR LE TOURNAGE SUR UNE MACHINE CNC DE AL7075-T6 EN UTILISANT L’ALGORITHME HYBRIDE TAGUCHI QPSO

RÉSUMÉ
Cet article combine la méthode Taguchi de réponse de surface (RSM) avec un algorithme d’optimisation par essaims particulaires pour prédire la rugosité de surface optimale d’une pièce de Al7075-T6 passant par l’usinage CNC. En premier lieu, les tableaux d’orthogonalité de Taguchi ont été appliqués pour déterminer les paramètres de coupe cruciaux : vitesse d’avancement ; angle de dépouille, profondeur de la coupe. Ensuite, la réponse de surface a été utilisée pour la construction du modèle prédictif de la rugosité de surface (\(R_a\), \(R_{\text{max}}\), et \(R_z\)). Finalement, le MOHQPSO avec mutation a été utilisé pour déterminer la rugosité optimale, et les conditions de coupe. Les résultats indiquent, en comparaison avec la non-optimisation, que la combinaison de la méthode Taguchi et l’algorithme d’optimisation multi-objectif par essaims particulaires (MOPSO) et la rugosité \(R_a\) utilisant MOHQPSO avec la solution optimale Pareto, ont été améliorés de 68.24, 59.31 et 33.80% respectivement. Ceci démontre que le modèle prédictif peut améliorer la qualité de tournage sur CNC de Al7075-T6.

Mots-clés : méthode Taguchi; RSM; MOHQPSO; tournage CNC; rugosité de surface.
1. INTRODUCTION

Aluminum Al7075-T6 is an age-hardened Al-Zn-Mg-Cu alloy with a strength comparable to many steels. When this type of alloy is subjected to heat treatment to produce high density, it exhibits exceptional plasticity after solution treatment. Moreover, the tensile strength of Al7075 T6 alloys subjected to tempering treatment is around 570 MPa [1], which is close to the hardness of medium carbon steel and has good fatigue strength. With its excellent resistance to corrosion and mechanical strength at low temperatures, Al7075-T6 is suitable for sports products, aircraft structural components, and moulds requiring high-intensity conditions and resistance to corrosion. However, although Al7075-T6 possesses high strength and hardness for machining operations, the influence of built-up edge (BUE) make controlling the surface roughness in turning machining difficult. Therefore, selecting the appropriate cutting parameters for turning Al7075-T6 plays a critical role in improving its roughness. Numerous previous studies have mentioned the surface roughness of cutting Al7075-T6. Basak and Goktas [2] used fuzzy theory to analyze the turning of Al7075-T6 for optimizing the cutting parameters involved in polishing processes. Bhushan et al. [3] employed a numerical method to analyze the surface roughness of Al7075-T6 in computer numerical control (CNC) turning. Chavoshi [4] employed an artificial neural networks and fuzzy theory to predict the parameter conditions of the CNC turning of Al7075 T6 for analyzing the relationships among the surface roughness, wear, and friction of the relief angle of tools. Choudhary and Chauhan [5] applied the response surface methodology (RSM) to determine the processability of Al7075-T6 and the minimal cutting force required to obtain the optimal roughness. In addition, Singh and Sodhi [6] employed the RSM to investigate the cutting parameters of the CNC turning of Al7020 to obtain the maximal material removal (MMR) and a lower roughness. Rao and Krishna [7] used a non-dominated Sorting Genetic Algorithm-II (NSGA II) for evaluating the surface roughness, MMR, and wire-electrode wear when performing electrical discharge machining (EDM). Subramanian et al. [8] used RSM to predict the surface roughness of Al7075-T651 work piece with high-speed steel end milling in terms of tool geometry. Vakondios et al. [9] studied the effects of tool angles on surface roughness for the milling of Al7075 T6 by using ball nose end mills. Kurt et al. [10] performed a grey relational analysis in the milling of Al7075-T651 to investigate and discuss the measurement errors in surface roughness.

According to the aforementioned studies, using artificial intelligence algorithms is a highly efficient method of improving the surface roughness in turning machining. However, there are only few studies conducted artificial intelligence algorithms to determine the optimal cutting parameters required for improving the surface roughness of Al7075-T6 in CNC turning processes. In this study, the Taguchi method was applied and an analysis of variance (ANOVA) was performed to determine the crucial cutting parameters related to the CNC turning of Al7075-T6. First, the spindle speed, feed rate, cutting depth, cutting length, tool nose radius, and tool relief angle were investigated to determine how they influence the degree of surface roughness and to determine the crucial cutting parameters for obtaining the optimal surface roughness. Subsequently, the RSM was applied to construct the models for predicting surface roughness including centre-line average roughness ($R_a$), maximum-height roughness ($R_{\text{max}}$), and ten-point height ($R_z$). Finally, a multi-objective hybrid quantum-behaved particle swarm optimization (MOHQPSO) method with mutation operator was employed to identify the appropriate parameter conditions along the Pareto-optimal front to determine the optimal predictive values of surface roughness.

2. DESCRIPTION OF SURFACE ROUGHNESS

Surface roughness indicates the degree to which the surface of a machined surface of a work piece is undulating. Machined surfaces typically exhibit short distances between two peaks or two valleys, which are indistinguishable to the naked eye. Numerous parameters can be employed to calculate the roughness through the measurement, including $R_a$, $R_{\text{max}}$, and $R_z$, as shown in Fig. 1.
3. MULTI-OBJECTIVE OPTIMIZATION MODEL

In practical applications, many optimization problems consider various objectives at the same time; therefore, there is no single solution to a multi-objective optimization problem, but rather a set of solutions, which are referred to as trade-off solutions. A multi-objective optimization problem is described in the following:

\[
\begin{align*}
\text{Min} & \quad R(x) = \{R_1(x), R_2(x), R_3(x), \ldots, R_m(x)\} \\
\text{s.t.} & \quad x^{(a)} \leq x \leq x^{(b)}
\end{align*}
\]

where solution \( x = [x_1, x_2, \ldots, x_k]^T \in D, \) \( D \) is the design parameter space, \( a \) and \( b \) are the lower limit and upper limit of each design parameter value. To identify trade-off solutions despite conflicts between objective functions, a superior method called the Pareto optimal method was used to obtain the trade-off solutions along the Pareto optimal frontier, which is called Pareto optimal solutions.

4. RESEARCH METHOD

4.1. Taguchi Method and Analysis of Variance

The Taguchi method [11, 12] is used to transform quality characteristics into a signal-to-noise ratio (S/N dB). This involves employing the S/N ratio to convert experimental results in an orthogonal array into a value for evaluation; this value is the quantity index of measuring qualities and is used to evaluate the degree of stability of processes and products. After the S/N ratios for each of the trials are determined, an analysis of variance (ANOVA) is used to determine the control parameters that are significant. The distinguished parameters in ANOVA are treated as optimal design parameters for creating a predictive model of quality characteristics. In this study, “the smaller-the-better” (STB) concept was used to determine the quality characteristic of the surface roughness. For STB, the more the better target approaches zero. Let \( y_e \) be the \( e \)th trial result and \( n \) the number of trials. The method is described as

\[
S/N (\text{dB}) = -10\log \left( \frac{1}{n} \sum y_e^2 \right)
\]
4.2. Response Surface Methodology
The Response Surface Methodology (RSM) is based on the suitability of empirical models and involves using a sequence of designed experiments to construct a predictive model consisting of linear or square polynomial functions [13]. It enables determining the relationships between specified ranges of design variables and one or more response variables. In this study, a second-order polynomial model with a central composite design (CCD) fitted to the experimental data was used to model the response surface. Let \( x_i \) and \( x_j \) are input variables that influence the response \( R \); \( \varepsilon \) is the error item. The second-order polynomial response surface can be expressed using

\[
R = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i^2 x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon
\]  

(3)

4.3. Description of Multi-objective Hybrid Quantum Behaved Particle Swarm Optimization
4.3.1. Overview of classical and quantum behaved particle swarm optimizations
Classical particle swarm optimization [14] (PSO) is a global search technique which was inspired by the social behaviour of flocks of birds and schools of fish. PSO identifies the most appropriate solution by considering the past behaviour of each particle and comparing it with the past and present behaviour of other particles in a group of particles that move simultaneously. After iterative calculation, most of the particles gradually move to the vicinity of the optimal solution of an entire group. The formula for the updating process is expressed as

\[
v_p(t+1) = w(t) \times v_p(t) + c_1 \times \text{rand}_1 \times (p_{\text{best}_p}(t) - x_p(t)) + c_2 \times \text{rand}_2 \times (g_{\text{best}}(t) - x_p(t))
\]  

(4)

\[
x_p(t+1) = x_p(t) + v_p(t+1)
\]  

(5)

\[
w(t) = w_{\text{max}} - \left[ (w_{\text{max}} - w_{\text{min}}) / (\text{iter}_{\text{max}}) \right] \times \text{iter}(t)
\]  

(6)

The current position and velocity vectors associated with each particle \( p \) are \( x_p(t) \) and \( v_p(t) \). \( t \) is the current iteration number. \( w_{\text{max}} \) and \( w_{\text{min}} \) are the maximal and minimal inertia weight factors in the range \([0, 1]\). \( \text{rand}_1 \) and \( \text{rand}_2 \) are the random values within \([0, 1]\), \( c_1 \) and \( c_2 \) are learning factors which are positive value, and \( w(t) \) is the dynamic weighting factor. \( P_{\text{best}_p}(t) \) is the best position of the \( p \)th particle, and \( g_{\text{best}}(t) \) is the best position among the individual (group best). However, the convergent speed of classical PSO decreases and the process easily stalls at a local optimal solution when the iterative algorithm proceeds to the late period. In 2004, Sun [15] proposed quantum behaved PSO (QPSO), which is an improvement of the classical PSO algorithm used in quantum mechanics. In a quantum space, the position and velocity of each particle cannot be simultaneously determined but can be depicted by a wave function \( \Phi(x, t) \). If each particle is described by the same wave function, then the probability density \( |\Phi|^2 \) of its position in the solution space can be described. QPSO algorithm is no velocity vector for each particle, and therefore, its search capability is superior to all presented PSO algorithms. The formulations of QPSO are described as follows:

\[
m_{\text{best}_p}(t) = \sum_{p=1}^{N} \frac{g_p(t)}{N}
\]  

(7)

\[
g_p(t) = \phi(t) \times p_{\text{best}_p}(t) + (1 - \phi(t)) \times g_{\text{best}}(t)
\]  

(8)

\[
x_p(t+1) = g_p \pm \alpha |m_{\text{best}_p}(t) - x_p(t)| \times \ln \left( \frac{1}{u(t)} \right)
\]  

(9)

\[
\alpha = (\alpha_{\text{max}} - \alpha_{\text{min}}) \times \frac{\text{iter}_{\text{max}} - t}{\text{iter}_{\text{max}}} + \alpha_{\text{min}}
\]  

(10)
where \( m \)\text{best} is defined as the mean of the \( p \)\text{best} positions of all particles, \( N \) is the number of all particles (population size), and the random variables \( u \) and \( \phi \) are the uniform probability distribution in range \([0, 1]\). \( \alpha \) is a design parameter called contraction-expansion coefficient, which can be tuned to control the convergence speed of the algorithms. This coefficient has a dynamic value that decreases from \( \alpha_{\text{max}} \) to \( \alpha_{\text{min}} \) with the number of iterations increases. The term \( g_p(t) \) is the best position of particle \( p \) at time \( t \).

### 4.3.2. MOHQPSO algorithm with mutation operator

The search area of each particle greatly depends on the particle best (\( p \)\text{best}) and global best (\( g \)\text{best}) whether the classical PSO or QPSO algorithm is applied. The diversity of a particle swarm markedly decreases during the early evolitional iterations; consequently, the solutions are easily trapped in a local optimal area. In this study, a broader search area was achieved by employing a mutation mechanism and using a real parameter in QPSO; this process is called hybrid quantum behaving particle swarm optimization (HQPSO).

As the effects of \( p \)\text{best} and \( g \)\text{best} in HQPSO can be reduced gradually, the number of cutting parameters is multiplied by a random value within the range \([0, 1]\) to identify the mutation performed for each particle. If the fitness value of a new particle mutated in HQPSO is superior to the current optimal fitness value, then this new particle is adopted as the new global optimal position. In this study, the mutation probability for each particle was set to 10%.

\[
x_{p,h}(t+1)_{\text{mut}} = x^{(a)}_{p,h}(t+1) + \text{rand} \times [x^{(b)}_{p,h}(t+1) - x^{(a)}_{p,h}(t+1)] \\
\]

where \( h = \text{ceil}(\text{rand}_3 \times k), k \in Z \).

The term “ceil (\( \cdot \)”), called the “ceiling function”, represents the lowest integer greater than or equal to the value of (\( \text{rand}_3 \times k \)), and \( h \) is the variable sequence position at which a mutation is obtained. \( \text{rand}_3 \) and \( \text{rand}_4 \) are random numbers within \([0, 1]\). In this study, \( k = 3 \) was conducted in the variable space \( x \); thus, \( h \) was an integral value within the range \([1, 3]\). Cutting conditions affect various characteristics of surface roughness; therefore, \( R_a \), \( R_{\text{max}} \), and \( R_z \) were simultaneously used as the fitness values for formulating a multi-objective optimization problem. The multi-objective hybrid quantum behaving particle swarm optimization (MOHQPSO) algorithm was used in this study. During the computing process, the MOHQPSO algorithm was designed to update the variables repeatedly according to Eqs. (7)–(11), until termination conditions were reached.

### 5. RESEARCH PROCESS

This paper proposes an integrated approach for investigating the effects of the cutting parameters and surface roughness \( R_a \), \( R_{\text{max}} \), and \( R_z \) in CNC turning of Al7075-T6, accordingly, identifying the optimal cutting parameters and predictive model for the turning process. A flow diagram of the proposed approach is shown in Fig. 2.

### 6. EXPERIMENTAL APPARATUS

As shown in Fig. 3, the experimental apparatus contained Fanuc O-T CNC turning and tungsten carbide tools, which were used in machining an Al 7075-T6 alloy of \( \phi 30 \) mm and \( 55 \) mm in length.

The feed rate was within the range 0.107 to 0.303 mm/rev; the spindle speed ranged from 1750 to 2550 rpm; the cutting depth of the tool ranged from 1.5 to 2.5 mm; the nose radii of the used tools were 1.5, 2, and 2.5 mm; and the relief angles of the tools were 8, 10, and 12°. The surface roughness was measured using a Sufcorder SE1200 machine.
7. RESULTS AND VERIFICATION

7.1. Determination of Crucial Cutting Parameters
This section presents an orthogonal array $L_{27}(3^{6})$ identified using the Taguchi method to determine the crucial cutting parameters, including the spindle speed, feed rate, end relief angle, tool nose radius, cutting length, and cutting depth, as shown in Table 1; this involved conducting 27 experiments in the CNC turning
Table 2. The $L_{27}(3^{27})$ Orthogonal array and output response.

<table>
<thead>
<tr>
<th>No.</th>
<th>Cutting Parameters (Factors)</th>
<th>$R_a$ ($\mu m$)</th>
<th>$R_{max}$ ($\mu m$)</th>
<th>$R_z$ ($\mu m$)</th>
<th>Average</th>
<th>S/N (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.107 1750 8 6 1.5 2.0</td>
<td>0.155</td>
<td>1.199</td>
<td>0.591</td>
<td>0.648</td>
<td>3.764</td>
</tr>
<tr>
<td>2</td>
<td>0.107 1750 8 6 2.0 2.5</td>
<td>0.114</td>
<td>0.565</td>
<td>0.390</td>
<td>0.356</td>
<td>8.963</td>
</tr>
<tr>
<td>3</td>
<td>0.107 1750 8 6 2.5 3.0</td>
<td>0.260</td>
<td>1.947</td>
<td>1.158</td>
<td>1.122</td>
<td>-0.997</td>
</tr>
<tr>
<td>4</td>
<td>0.107 2150 10 8 1.5 2.0</td>
<td>0.134</td>
<td>0.817</td>
<td>0.575</td>
<td>0.509</td>
<td>5.871</td>
</tr>
<tr>
<td>5</td>
<td>0.107 2150 10 8 2.0 2.5</td>
<td>0.100</td>
<td>0.528</td>
<td>0.339</td>
<td>0.322</td>
<td>9.834</td>
</tr>
<tr>
<td>6</td>
<td>0.107 2150 10 8 2.5 3.0</td>
<td>0.294</td>
<td>2.086</td>
<td>1.032</td>
<td>1.137</td>
<td>-1.118</td>
</tr>
<tr>
<td>7</td>
<td>0.107 2550 12 10 1.5 2.0</td>
<td>0.096</td>
<td>0.679</td>
<td>0.321</td>
<td>0.365</td>
<td>8.746</td>
</tr>
<tr>
<td>8</td>
<td>0.107 2550 12 10 2.0 2.5</td>
<td>0.115</td>
<td>0.523</td>
<td>0.383</td>
<td>0.340</td>
<td>9.362</td>
</tr>
<tr>
<td>9</td>
<td>0.107 2550 12 10 2.5 3.0</td>
<td>0.167</td>
<td>1.122</td>
<td>0.700</td>
<td>0.663</td>
<td>3.570</td>
</tr>
<tr>
<td>10</td>
<td>0.205 1750 10 10 1.5 2.5</td>
<td>0.100</td>
<td>0.519</td>
<td>0.379</td>
<td>0.333</td>
<td>9.560</td>
</tr>
<tr>
<td>11</td>
<td>0.205 1750 10 10 2.0 3.0</td>
<td>0.151</td>
<td>1.019</td>
<td>0.565</td>
<td>0.578</td>
<td>4.756</td>
</tr>
<tr>
<td>12</td>
<td>0.205 1750 10 10 2.5 2.0</td>
<td>0.239</td>
<td>1.138</td>
<td>0.769</td>
<td>0.715</td>
<td>2.910</td>
</tr>
<tr>
<td>13</td>
<td>0.205 2150 12 6 1.5 2.5</td>
<td>0.130</td>
<td>0.769</td>
<td>0.517</td>
<td>0.472</td>
<td>6.521</td>
</tr>
<tr>
<td>14</td>
<td>0.205 2150 12 6 2.0 3.0</td>
<td>0.493</td>
<td>2.374</td>
<td>1.717</td>
<td>1.528</td>
<td>-3.683</td>
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<td>15</td>
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<td>0.138</td>
<td>0.964</td>
<td>0.511</td>
<td>0.538</td>
<td>5.390</td>
</tr>
<tr>
<td>16</td>
<td>0.205 2550 8 8 1.5 2.5</td>
<td>0.302</td>
<td>2.294</td>
<td>0.971</td>
<td>1.189</td>
<td>-1.504</td>
</tr>
<tr>
<td>17</td>
<td>0.205 2550 8 8 2.0 3.0</td>
<td>0.145</td>
<td>1.216</td>
<td>0.648</td>
<td>0.670</td>
<td>3.483</td>
</tr>
<tr>
<td>18</td>
<td>0.205 2550 8 8 2.5 2.0</td>
<td>0.786</td>
<td>5.720</td>
<td>3.324</td>
<td>3.277</td>
<td>-10.309</td>
</tr>
<tr>
<td>19</td>
<td>0.303 1750 12 8 1.5 3.0</td>
<td>0.145</td>
<td>0.598</td>
<td>0.500</td>
<td>0.414</td>
<td>7.653</td>
</tr>
<tr>
<td>20</td>
<td>0.303 1750 12 8 2.0 2.0</td>
<td>0.215</td>
<td>1.276</td>
<td>0.805</td>
<td>0.765</td>
<td>2.323</td>
</tr>
<tr>
<td>21</td>
<td>0.303 1750 12 8 2.5 2.5</td>
<td>0.161</td>
<td>0.978</td>
<td>0.632</td>
<td>0.590</td>
<td>4.578</td>
</tr>
<tr>
<td>22</td>
<td>0.303 2150 8 10 1.5 3.0</td>
<td>0.213</td>
<td>1.196</td>
<td>0.622</td>
<td>0.677</td>
<td>3.388</td>
</tr>
<tr>
<td>23</td>
<td>0.303 2150 8 10 2.0 2.0</td>
<td>0.438</td>
<td>1.782</td>
<td>1.543</td>
<td>1.254</td>
<td>-1.968</td>
</tr>
<tr>
<td>24</td>
<td>0.303 2150 8 10 2.5 2.5</td>
<td>0.438</td>
<td>2.195</td>
<td>1.250</td>
<td>1.294</td>
<td>-2.241</td>
</tr>
<tr>
<td>25</td>
<td>0.303 2550 10 6 1.5 3.0</td>
<td>0.129</td>
<td>0.804</td>
<td>0.462</td>
<td>0.465</td>
<td>6.651</td>
</tr>
<tr>
<td>26</td>
<td>0.303 2550 10 6 2.0 2.0</td>
<td>0.273</td>
<td>1.490</td>
<td>0.806</td>
<td>0.856</td>
<td>1.347</td>
</tr>
<tr>
<td>27</td>
<td>0.303 2550 10 6 2.5 2.5</td>
<td>0.463</td>
<td>2.652</td>
<td>1.439</td>
<td>1.518</td>
<td>-3.625</td>
</tr>
</tbody>
</table>

All results average 0.837 3.082

Fig. 4. Main effect plots for response.

As shown in Table 2 and Figs. 5(a–f), the optimal combination was A2B3C1D2E3F1. According to the ANOVA, Factors A, C, and E were greater than $F_{0.1,6,27} = 2.300$ at a 90% confidence level, demonstrating the optimal levels and transform the target value in each experiment of factors into an S/N ratio of Al7075-T6 to identify the optimal levels and transform the target value in each experiment of factors into an S/N ratio.
Table 3. The ANOVA results.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DOF</th>
<th>SS</th>
<th>Var</th>
<th>F</th>
<th>ρ%</th>
<th>Confidence</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>68.418</td>
<td>34.209</td>
<td>2.427</td>
<td>10.673</td>
<td>&gt;90%</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>42.452</td>
<td>21.226</td>
<td>1.506</td>
<td>6.622</td>
<td>&lt;75%</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>120.767</td>
<td>60.383</td>
<td>4.285</td>
<td>18.839</td>
<td>&gt;95%</td>
<td>Yes</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>18.511</td>
<td>9.255</td>
<td>0.657</td>
<td>2.888</td>
<td>&lt;75%</td>
<td>No</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>160.516</td>
<td>80.258</td>
<td>5.695</td>
<td>25.04</td>
<td>&gt;95%</td>
<td>Yes</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>33.069</td>
<td>16.534</td>
<td>1.173</td>
<td>5.159</td>
<td>&lt;75%</td>
<td>No</td>
</tr>
<tr>
<td>Error</td>
<td>14</td>
<td>197.306</td>
<td>14.093</td>
<td></td>
<td></td>
<td>At least 90% confidence</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>641.038</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 4. Level for design variables in actual and coded value.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Level</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁</td>
<td>–1.682</td>
<td>–1</td>
</tr>
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<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6818</td>
</tr>
<tr>
<td>x₂</td>
<td>Feed rate (mm/rev)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.205</td>
</tr>
<tr>
<td>x₃</td>
<td>Relief angle (°)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>x₄</td>
<td>Cutting depth (mm)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.5</td>
</tr>
</tbody>
</table>

Fig. 5. Predictive models of \( R_a \).

that the feed rate, tool relief angle, and cutting depth exerted a significant effect. The cutting depth was the most crucial factor. In other words, the combination of the Taguchi method in conjunction with the ANOVA markedly improved the quality of the surface roughness when the cutting depth was slightly adjusted.

7.2. Constructions of Predictive Model from RSM

The experimental design involved constructing 20 sets of coded conditions and raw experimental data in RSM, according to Eq. (3) and Table 4. The mathematical predictive models with three-level of CCD were
expressed as follows:

\[R_a(x) = 0.17426 + 0.03939x_1 - 0.09361x_2 - 0.01638x_3 + 0.03118x_1^2 + 0.03206x_2^2 + 0.02835x_3^2 \]

\[R_{\text{max}}(x) = 1.01573 + 0.32269x_1 - 0.64265x_2 - 0.12371x_3 + 0.19924x_1^2 + 0.19977x_2^2 + 0.23777x_3^2 \]

\[R_c(x) = 0.57875 + 0.15661x_1 - 0.34859x_2 - 0.08481x_3 + 0.13862x_1^2 + 0.15011x_2^2 + 0.08294x_3^2 \]

\[+ 0.03875x_1x_2 - 0.052x_1x_3 + 0.04125x_1x_2 \]  

\[R_{\text{max}}(x) = 1.01573 + 0.32269x_1 - 0.64265x_2 - 0.12371x_3 + 0.19924x_1^2 + 0.19977x_2^2 + 0.23777x_3^2 \]

\[R_c(x) = 0.57875 + 0.15661x_1 - 0.34859x_2 - 0.08481x_3 + 0.13862x_1^2 + 0.15011x_2^2 + 0.08294x_3^2 \]

\[+ 0.03875x_1x_2 - 0.052x_1x_3 + 0.04125x_1x_2 \]  

(12)

The 3-D surface diagrams and 2-D contour maps of the RSM predictive models from Eqs. (12) are shown in Figs. 5–7. Figures 5(a) and 5(d), 6(a) and 6(d), and 7(a) and 7(d) show that \(R_a\), \(R_{\text{max}}\), and \(R_c\), respectively, were low when a cutting depth of 1.5 mm was used, the feed rates were low, and the relief angles were large. Figures 5(b) and 5(e), 6(b) and 6(e), and 7(b) and 7(e) show that \(R_a\), \(R_{\text{max}}\), and \(R_c\), respectively, were low when a relief angle of 12° was used, the feed rates were low with a large end relief angle. \(R_{\text{max}}\) is low when the feed rate of 0.03–0.1 mm/rev and the cutting depth of 1.2–1.8 mm were used. Figures 5(c) and 5(f), 6(c) and 6(f), and 7(c) and 7(f) show that \(R_a\), \(R_{\text{max}}\), and \(R_c\), respectively, were low when a feed rate of 0.107 mm/rev and a cutting depth of 1.5 mm with a larger end relief angle were applied. Selecting any cutting depth can cause \(R_a\) to get lower values when the relief angle was greater than 14°. In addition, the cutting depth between 1 and 2.2 mm caused \(R_c\) to be low when the small relief angle was used in the machining process.

7.3. Pareto Optimal Solutions and Experimental Verification

As shown in Eqs. (7)–(11), the mathematical predictive models were optimized using MOHQPSO algorithm to determine the optimal values of cutting parameters. Because the objective was attaining a minimal surface roughness according to Eq. (12), the MOHQPSO algorithm was used to solve this multi-objective minimization problem. The multi-objective optimization functions were shown in Eq. (13). The initial num-
ber of particles in MOHQPSO was set to 800, and the contraction expansion coefficients $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$ were set to 0.5 and 1.0, respectively. Each particle represented a combination of three cutting parameters. After 1000 iterations of the MOHQPSO algorithm, the 3-D Pareto optimal front was determined.

$$\text{Min } R(x) = \{R_a(x_1, x_2, x_3), R_{\text{max}}(x_1, x_2, x_3), R_c(x_1, x_2, x_3)\}$$

s.t. $0.009 \text{ mm/rev} \leq x_1 \leq 0.205 \text{ mm/rev}$

$0.5 \text{ mm} \leq x_2 \leq 2.5 \text{ mm}$

$8^\circ \leq x_3 \leq 16^\circ$ (13)

Figures 8(a) and 8(b) show the results of classical MOPSO and MOHQPSO. Figures 9(a) and 9(b) display the 2-D Pareto optimal frontier from the projection onto two objectives of Fig. 8(b). Compared with non-optimization, after the Taguchi method and classical MOPSO (the same contraction-expansion coefficients, iterations and numbers of particles as MOHQPSO) were applied in the CNC turning of Al7075-6T alloy,
the surface roughness \( R_a \) achieved using MOHQPSO was reduced by 68.24, 59.31 and 33.80\%; \( R_{\text{max}} \) was reduced by 65.28, 54.07, and 16.59\%; and \( R_z \) was reduced by 58.95, 54.28, and 41.71\% as shown in Table 5.

Table 6 shows the experimental results with five groups of optimal cutting conditions. Compared with the results of MOHQPSO, the mean squared errors (MSE) in \( R_a \), \( R_{\text{max}} \), and \( R_z \) were 0.0097, 0.0348, and 0.0364 \( \mu \)m, respectively. Figure 10 shows the results of experimental verification when non-optimization, the Taguchi method, or MOHQPSO was applied. Above all, Figs. 10(c)–10(g) display that the roughness have larger variations with the cutting depth adjusted. The reason is that the cutting depth is identified as the most significant parameter controlling the surface roughness. The MSE of \( R_a \) in Table 6 was extremely small. Because the \( R_a \), \( R_{\text{max}} \) and \( R_z \) are different types of formulations with different definitions, the errors of \( R_{\text{max}} \) and \( R_z \) were amplified with different measuring positions of the roughness curve. The results showed that applying MOHQPSO algorithm effectively reduced surface roughness and enabled accurately predicting the surface roughness in the CNC turning of Al 7075-T6 alloy.

8. CONCLUSION

In this study, the Taguchi method and RSM were first applied to develop a second-order mathematical model for predicting surface roughness values for known values of the tool relief angle, cutting depth, and
feed rate in the CNC turning of Al7075-T6 alloy. The direct and interaction effects of the input parameters were analyzed using the predictive models. The mathematical predictive models were optimized using MOHQPSO to attain the optimal surface roughness according to trade-off solutions. The predicted values of roughness were consistent with the observed values. The findings of this study are valuable for improving the machining performance and product qualities because they enable achieving optimal surface roughness and reducing the cost of turning operations.

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