A NUMERICAL AND EXPERIMENTAL INVESTIGATION OF COALESCENCE BETWEEN CYLINDRICAL HOLES

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ABSTRACT
A sequential digital image technique was employed to detect the onset of coalescence between pairs of holes in tensile specimens. The coalescence event was induced between cylindrical holes that were positioned with various spacing and ligament orientations. Using finite-element analysis as a framework, the coalescence predictions of a classic micromechanical model were compared with the experimental coalescence results. It was found that the predicted strains at coalescence could be significantly improved by accounting for local work-hardening in the ligament region between the two holes. Digital image correlation was used to extract strain values from the digital image record, for both the far-field and the intervoid ligament.

Keywords: plastic limit-load; void coalescence; ductile fracture.

INVESTIGATION NUMÉRIQUE ET EXPÉRIMENTALE DE LA COALESCENCE ENTRE DES TROUS CYLINDRIQUES

RÉSUMÉ
Une technique d’imagerie digitale séquentielle a été employée pour détecter l’apparition de coalescence entre des paires de trous sur les spécimens de traction L’apparition de coalescence a été induite entre les trous cylindriques qui étaient positionnés en un espacement varié aussi bien que les orientations de ligaments. En utilisant la méthode des éléments finis comme cadre, les prédictions de coalescence d’un modèle micromécanique classique furent comparées avec les résultats des expériences de coalescence. On a trouvé que les tensions prévues de coalescence pourraient être significativement améliorées en comptant sur le durcissement (écrouissage) dans la région de ligament entre les deux trous. Une corrélation de l’image digitale a été utilisée pour extraire les valeurs de déformation de l’enregistrement des images digitales pour le champ dans la zone proche ainsi que pour les ligaments entre les trous.

Mots-clés : charge limite plastique ; coalescence du vide ; rupture ductile.
1. INTRODUCTION

Ductile fracture in porous ductile materials is due to the nucleation and growth of micro-voids that coalesce to form micro-cracks that link-up to cause rupture. The voids may be pre-existing from material processing, or they may nucleate from second-phase particles via cracking or interface separation during deformation. The micro-void coalescence process is a complex function of the void geometry, the local stress state, and the relative orientation of the intervoid ligament with respect to the principal loading direction [1–8].

Over the last decade, ‘damage percolation’ multi-scale material models have been developed that employ measured second-phase particle distributions to capture void nucleation, growth and coalescence within the actual microstructure. Percolation models use micromechanical sub-models to predict void initiation and evolution during deformation and thus capture the macroscopic influence of the voids on the material response [1–3]. Real microstructures contain particle and void clusters of various orientations relative to the principal loading direction that serve as preferential sites for premature strain localization and fracture. To consider these particle fields, it is necessary that the sub-models for void evolution and coalescence are not based upon assumed cluster configurations, such as a periodic distribution of voids orthogonal to the major straining direction. This motivates the present experimental investigation of angled intervoid ligaments in relation to the void coalescence event, and the experimental evaluation of Thomason’s [4] coalescence model in the situation of arbitrary ligament orientations.

Void coalescence is a difficult phenomenon to observe experimentally since it occurs abruptly on the micro-scale. This has motivated the use of contrived materials with voids modelled as cylindrical drill-holes having length-scales on the order of millimetres [5–9]. Recently, laser-machined holes have been employed to create coalescence experiments with holes on the order of micrometres in diameter [10, 11]. Three-dimensional model materials have also been used, where X-ray tomography enabled the in-situ study of nucleation, growth, and coalescence of spherical microvoids encased in a matrix [12].

The present work uses a direct and inexpensive approach to: (i) experimentally observe incipient coalescence between drilled holes, (ii) demonstrate the influence of the intervoid ligament angle on coalescence, and, (iii) to experimentally and quantitatively assess the ability of Thomason’s cylindrical voids model [4] to predict coalescence in the situation of an arbitrary ligament orientation. This study will provide a valuable benchmark for the development of future coalescence models for cylindrical holes subjected to general stress states and improve the modelling of ductile fracture when two-dimensional representative elements of particle fields are considered [1–3].

2. THEORETICAL BACKGROUND

Thomason [4] pioneered the plastic limit-load (PLL) void coalescence model and elucidated that there is a competition between stable homogenous deformation and localized unstable deformation in the microstructure of a porous-ductile material during plastic flow. Initially, the influence of the voids is negligible and the material deforms homogeneously. As deformation continues and the voids grow, the energy required for transition to a localized deformation mode within the intervoid ligaments decreases. The onset of localization or incipient coalescence (IC) occurs when the energy required to sustain homogeneous deformation is equal to the energy required to achieve localized deformation. This point is known as the plastic limit-load and is followed by ligament failure as the neighboring voids coalesce into a larger void or micro-crack. Thomason [4] developed his PLL condition for both cylindrical and ellipsoidal voids. Eq. (1) is Thomason’s PLL condition for cylindrical voids where $W$ is the void aspect ratio (ratio of $R_y$ to $R_x$), $\chi$ is the ligament size ratio (ratio of $R_x$ to $S$), $\bar{\sigma}$ is the current average yield stress of the material, and $\psi$ is the angle of the maximum principal stress relative to the material ligament ($\psi = \pi/2 - \theta$, where $\theta$ is the so-called ‘ligament angle’). Refer to Fig. 1 for further explanation of the geometry under consideration. The onset of void
coalescence occurs when the following constraint is satisfied:

$$\frac{\sigma_1}{\bar{\sigma}} \geq \frac{2}{\sqrt{3}} \left( \sqrt{1 + \frac{1}{4\tan^2\psi} + \chi^{-1}} \right) \left( 1 - \frac{\pi}{4} \chi \right).$$

(1)

Note that Eq. (1) is the version of Thomason’s cylindrical voids model that inherently accounts for the ligament angle [4]. The Thomason [4] plastic limit-load model has been well received in the literature with notable extensions by Pardoen and Hutchinson [13], who accounted for matrix hardening; Benzerga [14], who dealt with penny-shaped voids; Fabregue and Pardoen [15], who considered the influence of a secondary population of voids, and Tekoglu et al. [16], who proposed an extension of the PLL model to deal with combined tension and shear.

The left-hand side (LHS) of the plastic limit-load model in Eq. (1) is the ratio of the applied principal stress to the matrix flow stress and is a measure of the ‘homogeneous’ deformation within the material. Conversely, the right-hand side (RHS) of Eq. (1) describes ‘localized’ deformation and is a function of the microstructure geometry via the void shape and spacing. The RHS is alternatively known in the literature as the ‘plastic constraint factor’. The operation of the PLL model is best illustrated in Fig. 2, which shows the evolution of the RHS and LHS of the PLL model in Eq. (1) for an initial ligament size ratio of $\chi = 0.4$. The LHS, or stress ratio, remains essentially constant while the RHS decreases as the voids elongate into prolate cylinders and move closer together, increasing the void spacing ratio. Coalescence occurs when the plastic constraint factor reaches the same value as the LHS for homogenous deformation. The influence of strain-hardening in the ligament initially counteracts the influence of the stress concentration due to the presence of the voids, but eventually, the hardening rate decreases at higher plastic strains and coalescence ensues.

3. CYLINDRICAL-HOLE COALESCENCE EXPERIMENTS

3.1. Material Characterization

A series of tensile experiments were performed in order to study the impact of the ligament angle on the far-field strain required to cause coalescence. Holes drilled in 12.7 mm thick, commercially pure aluminum plate were used to model the micro-voids that are encountered in the microstructure of porous-ductile materials. Commercially pure aluminum (AA1100) was selected in an attempt to mitigate the influence of

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Plot showing the intersection of the RHS and LHS of the PLL model for an initial ligament size ratio of 0.4, ligament angle of zero degrees, and cylindrical voids having an initial aspect ratio of one. The location of intersection denotes incipient coalescence. This chart is based on finite-element results and considers a hardening material.

### Table 1. Chemical composition of the commercially pure aluminum alloy, AA1100 (units of % weight).

<table>
<thead>
<tr>
<th></th>
<th>Cu</th>
<th>Mn</th>
<th>Zn</th>
<th>Fe</th>
<th>Si</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA1100</td>
<td>0.13</td>
<td>0.01</td>
<td>0.01</td>
<td>0.51</td>
<td>0.09</td>
<td>balance</td>
</tr>
</tbody>
</table>

Second-phase particles and inclusions on the coalescence mechanism between the drilled holes since such particles are void nucleation sites that can prematurely weaken the ligament and result in a void-sheeting coalescence mode [17]. Certainly, some measure of void nucleation will occur in all materials as there will always be some defects, and nucleation can occur within interstitial-free materials at the grain boundaries. Nevertheless, the commercially pure aluminum has a very clean microstructure and provides a reasonable approximation of a homogeneous matrix material. Table 1 shows the microstructural constituents of the aluminum used.

The flow stress relation of the pure aluminum was obtained from uniaxial tensile tests. To minimize the influence of anisotropy on the test specimens, all of the specimens in this study were fabricated with their longitudinal direction aligned with the rolling direction of the sheet. A Voce [18] hardening law was employed to characterize the flow stress of the aluminum and is expressed as

\[
\bar{\sigma} = \sigma_y - (\sigma_y - \sigma_s) \exp\left(-\alpha (\varepsilon^p)^\beta\right),
\]

where \(\varepsilon^p\) is the equivalent plastic strain, and the fitting parameters in Eq. (2) are: \(\sigma_y = 62 \text{ MPa}, \sigma_s = 120.6 \text{ MPa}, \alpha = 10.9\) and \(\beta = 0.9056\). A comparison of the experimental flow stress and Eq. (2) is presented in Fig. 3.

### 3.2. Specimen Geometry

A total of nine different specimen geometries were considered in the present study: three ligament size ratios (\(\chi = 0.3, 0.4, 0.5\)) with three separate ligament angles (\(\theta = \theta^\circ, 25^\circ, 45^\circ\)). For each geometry, six specimens were fabricated and tested until failure for a total of 54 tests conducted. Figure 4 is an example of the tension specimen geometry used. The naming convention for the specimens is demonstrated by the
Fig. 3. Experimental flow stress curve in the rolling direction and its comparison with Eq. (2).

Table 2. Drill bits used to create the holes in the 12.7 mm thick AA1100 specimens (L = 1.8 mm).

<table>
<thead>
<tr>
<th>Nominal Radius (mm)</th>
<th>Calculated Radius (mm)</th>
<th>Actual drill-bit Radius (mm)</th>
<th>Actual χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.387</td>
<td>0.394</td>
<td>0.305</td>
</tr>
<tr>
<td>0.4</td>
<td>0.603</td>
<td>0.596</td>
<td>0.395</td>
</tr>
<tr>
<td>0.5</td>
<td>0.905</td>
<td>0.889</td>
<td>0.491</td>
</tr>
</tbody>
</table>

following example: X04_25Deg_2, signifying a ligament size ratio of $\chi = 0.4$, a ligament angle $\theta = 25^\circ$, and a specimen identification number between one and six, 2 (to distinguish the specimen from other repetitions of the same geometry).

When experimentally investigating void/hole coalescence, it is important that the stress state within the ligament be representative of the stress state between two microvoids within the material. For this reason, care must be taken to ensure that a situation of plane stress does not arise since the stress state between two coalescing voids is closer to one of plane strain. Thomason [4] suggested that to create meaningful coalescence experiments, the predominant stress state between the modeled voids should follow the assumption of plane strain with a specimen thickness of at least five times greater than the hole spacing. Consequently, the ratio of the sample thickness to the hole spacing was kept constant at a value of $t/L = 7$ for all of the specimens in the present work. The required hole diameters to create three specified ligament size ratios were calculated based on $t/L = 7$ and a material thickness of 12.7 mm. Table 2 compares the nominal and actual ligament size ratio ($\chi$) values. Consult the Appendix for more information concerning specimen preparation and the digital image correlation (DIC) strain measurement system employed during the experiments.

3.3. Experimental Identification of Coalescence

The onset of coalescence was identified experimentally by visually reviewing the digital image record of each test. A frame-rate of one frame every six seconds was found to be sufficient to capture the transition from diffuse to localized deformation for quasi-static loading. The image corresponding to the transition between deformation modes was visually identified and the strain-distribution obtained using the DIC software using a virtual extensometer with a gauge length of 25 mm for the far-field strain measurements.
The initial ligament length was held constant over all the experiments at $L = 1.8$ mm. The material thickness was constant at $t = 12.7$ mm. The hole sizes varied between three designated diameters (see Table 2).

Fig. 4. Example tension specimen (units in millimeters, magnification of the hole-area is not to scale). The initial ligament length was held constant over all the experiments at $L = 1.8$ mm. The material thickness was constant at $t = 12.7$ mm. The hole sizes varied between three designated diameters (see Table 2).

The experimental identification of IC is difficult; however, the present work demonstrates the digital image technique for witnessing the event. In previous work, using X-ray tomography, Weck et al. [11] used ligament fracture as an indication of the coalescence event [11], but, in reality, the two events are unique and occur separately. The onset of coalescence (the event that the PLL model predicts) occurs prior to and is separate from the subsequent ligament failure event (see Fig. 6). The assumption that the two events are separated by a negligibly small strain increment is reasonable but not ideal. Weck et al. [11] recognized this, and reported that ideally one would compare the PLL model predictions to the actual instance of IC but that it was too difficult to consistently pinpoint IC in their experiments. The technique used in the present work, while simplistic, is an effective and repeatable method for experimentally observing the onset of coalescence in a model material containing drilled holes.

To be clear, the DIC system was used for the following two tasks: (i) finding the far-field strain value corresponding to the IC image, and (ii) comparing the vertical far-field strain and the vertical ligament strain values, enabling results like those displayed in Fig. 7. All other calculations were performed using finite-element (FE) simulation results. It could be argued that an extensometer and a camera could replace the use of DIC in this study, but that approach would introduce the requirement of syncing the extensometer readings with the image history. In addition, experimental comparison between the far-field strain values and the ligament strain values (e.g. Fig. 7) would be lost because extensometers supply relative displacement measurements between only two predefined points.

4. FINITE-ELEMENT SIMULATIONS

The commercial software *ABAQUS* [19] was used to simulate the nine unique uniaxial void coalescence experiments (three ligament angles for each of the three ligament size ratios). The FE results were used...
Fig. 5. FE model having ligament size ratio $\chi = 0.5$ and ligament angle $= 25^\circ$ (a) modeled part with boundary conditions (b) mesh around the holes.

to validate the experimental work performed and to provide a framework for evaluating the PLL models at each strain increment of deformation. The objective of the FE simulations was to determine at what far-field strain the PLL models predict coalescence of the drilled holes. The objective of the experiments was to determine the actual strain values at IC and failure. By post-processing the finite-element simulation data, the instantaneous values for the local stress state in the ligament (the flow stress in the ligament was taken from an element at the void edge, where the stress is maximum [15]), the void aspect ratio, and the ligament size ratio were extracted and used to evaluate the coalescence condition of Eq. (1).

A typical, finite-element model of the test geometry and its corresponding mesh are shown in Fig. 5. A structured mesh of 30,000 eight-node brick elements with reduced integration was found to remove mesh sensitivity and obtain a converged solution. A symmetry boundary condition through the thickness of the sample was employed to reduce the size of the FE model. A displacement boundary condition was applied to the top surface of the test sample and the bottom surface of the sample was fixed to mimic the grip of the Instron tensile testing machine. Using nodal displacements provided by the FE model, the void aspect ratio and ligament size ratio were calculated at each increment of the simulation. The far-field loading and ligament flow stress were also obtained from the FE model at each time increment during deformation. The PLL models were evaluated for coalescence at each time-step, and the far-field vertical strain at which the coalescence condition was satisfied was recorded. The predicted or theoretical coalescence strains obtained using the PLL models were then compared to the experimentally determined coalescence strains obtained using the DIC system.
Fig. 6. Load vs. strain history for the specimen X04_25Deg_1. The left-most diamond marker indicates IC, and the second diamond marker indicates visible ligament fracture. The digital images are from the DIC history of X04_25Deg_1 and show the state of the specimen at the instant specified by the accompanying arrow.

Fig. 7. Comparing the vertical strain in the ligament to the vertical far-field strain for three ligament size ratios while holding ligament angle constant. The solid-circle markers indicate IC. The end of each curve represents complete ligament fracture. The DIC system enabled this experimental comparison between the ligament strain values and the far-field strain values.

5. RESULTS AND DISCUSSION

5.1. Experimental Results

A load-displacement history of one of the tensile specimens is shown in Fig. 6 and illustrates the load drop that occurs at the onset of coalescence and the second load drop that coincides with ligament fracture. After the ligament fractures, the holes are connected by a macro-crack that propagates throughout the material.
resulting in complete fracture of the cross-section. Note that in this study, the objective was to predict the onset of localization and not the subsequent crack propagation. Fracture mechanics techniques would be required to model the crack propagation stage.

At the moment of IC, deformation completely localizes in the ligament. The inability of the ligament to further work-harden and the reduction of area due to necking cause a decrease in the load required for deformation of the specimen. This is demonstrated by the commencement of a negative slope in Fig. 6 (specimen geometry X04_25Deg_1) at a strain of 0.152. At a strain value of 0.175, the slope of the load vs. strain curve abruptly changes again, marking the commencement of the macro-crack propagation process and rupture of the tensile sample.

Figure 7 shows the DIC measured strain histories for three ligament size ratios while holding ligament angle constant. The vertical strain in the ligament is plotted against the vertical strain in the far-field. The exponential trend demonstrates the advanced strain hardening process experienced by the ligament compared to the far-field, even prior to IC. Strain concentration in the inter-void ligament was stronger for large ligament size ratios (‘closer’ holes).

The far-field principal strain was used as an indication of the extent of deformation and was the main independent variable used in the analysis of the numerical and experimental results and the comparison between the two. Figure 8 displays the mean far-field strain at the moment of IC for the specimen geometries considered in the experimental work. The 95% confidence interval for each mean is shown using interval bars. For a low ligament size ratio of \( \chi = 0.3 \) (large spacing between the holes), the ligament angle has less influence on the strain at IC. This is likely due to the fact that shear concentration is more severe between holes that are closely spaced (\( \chi = 0.4, 0.5 \)). At higher ligament size ratios (\( \chi = 0.4, 0.5 \)), the ligament angle has a pronounced influence on the IC strain; the IC strain increases with ligament angle. If the ligament angle is held constant and the ligament size ratio is increased, the strain required for IC drops.

As expected, the situation most susceptible to IC by internal necking is when the ligament is perpendicular to the applied load vector. As the ligament angle is increased, a progressively smaller component of the load
Fig. 9. Screenshots of X05_45Deg FE simulation: (a) at instant of IC (c) several steps into the post-IC regime. Notice the intense shearing, hole rotation, and hole flattening. Experimental digital images for same geometry: (b) at the onset of coalescence (d) at ligament failure. Notice the qualitative agreement between (a) and (b), and (c) and (d).

vector is perpendicular to the ligament, making IC by internal necking progressively less tenable (reflected by \( \tan^2 \psi \) in Eq. 1). However, altering the ligament orientation introduces shear that weakens the ligament and promotes coalescence via ligament shearing or combined tension and shear. The influence of combined tensile and shear stress states on coalescence has received recent attention in the literature and is an active area of research [20–23]. The experimental results shown in Fig. 8 highlight the importance of further study in the area of modeling void coalescence under combined tension and shear.

5.2. Comparison of Experimental and FE Void Evolution Trends
As shown by Fig. 9, the digital image history for X05_45Deg shows the same trends as the FE simulation for the same specimen geometry. It is interesting to notice the influence of the shear stress introduced by the slanted ligament. The mesh distortion in (a) and (c), and the angled fracture surface and slip in (d) show the strong influence of the combined tension and shear stress state.

5.3. Comparison of the Experimental and Predicted Coalescence Strains
Figure 10 compares the predictions of Thomason’s model (Eq. 1) to the experimental results. In keeping with the experimental results, Thomason’s model shows that increasing ligament angle serves to increase the strain required for coalescence. While the general trends are similar, the difference between the predicted and actual values is high. The greatest difference occurs for a ligament size ratio of 0.5, where the model mistakenly predicts coalescence of the holes very early in the plastic deformation stage.

5.4. Adjusting Thomason’s model
Figure 10 demonstrates poor correlation between Eq. (1) and the IC results. The deviation could be partially due to the lack of consideration of unique flow stress in the ligament by Eq. (1). Scheyvaerts et al. [23], among other contributions, proposed that ‘the magnitude of the stress associated with the localized mode of plastic yielding in the intervoid ligament is, in strain hardening materials, controlled by the local value of the current yield stress at the most deformed location of the ligament, instead of the current average yield stress’. Based on this proposition by Scheyvaerts et al. [23], the average yield stress \( \sigma \) in Eq. (1) is replaced by the local yield stress in the ligament \( \sigma_{lig} \) (see Eq. 3). The current yield stress at the most deformed location of the ligament was obtained from the FE simulation results. Eq. (3) is herein referred to as the ‘modified Thomason’ model. Figure 7 concurs with the proposition of Scheyvaerts et al. [23], bearing witness to significant strain concentration in the ligament (and therefore a raised flow stress) prior
Fig. 10. Comparison between the PLL predictions (Eq. 1) and the experimental IC strain values.

Fig. 11. Comparison between the experimental results and the PLL predictions after modifying the LHS of Eq. (1) to use the ligament flow stress in the denominator.

to coalescence, when compared to the far-field.

\[
\frac{\sigma_1}{\sigma_{\text{lig}}} \geq \frac{2}{\sqrt{3}} \left( \sqrt{1 + \frac{1}{4 \tan^2 \psi \chi^{-1}}} + \frac{\chi^{-1}}{4W} \right) \left( 1 - \frac{\pi}{4} \chi \right)
\]  

(3)

Equation (3) shows good improvement over Eq. (1), demonstrated by Fig. 11. The very early coalescence prediction for \( \chi = 0.5 \) is now averted. Using the ligament flow stress value instead of the average flow stress value effectively decreases the LHS ratio of the PLL equation, delaying the intersection of the two functions in Fig. 2, consequently delaying the prediction of IC. This effectively shifts the LHS of Eq. (1) in Fig. 2, downward. The percent differences between the experimental results and the IC predictions of
Eq. (3) range from 9 to 39%, with an average percent difference of 25%. When Weck et al. [11] compared their experimental results to Thomason’s cylindrical voids model they reported percent difference values of 34 to 66%, with an average percent difference of 45%. They cited that two causes for the discrepancy were: (i) the difference in the definition of incipient coalescence between their work and Thomason’s (as discussed in Section 3.3), and (ii) not accounting for material hardening (they used an Al-Mg alloy, AA5052). Both of these issues have been addressed in the present work, resulting in lower discrepancy (on average, 20% lower) between the model and the experimental results. In the present work, the definition of coalescence that was employed was the same definition as that of Thomason [4]. IC was captured using high-resolution digital images. In addition, material hardening was accounted for, specifically, the hardening behaviour of the intervoid ligament. The remaining disagreement between the predictions of the modified Thomason model and the experimental results is most likely due to the fact that the influence of shear is neglected by Eqs. (1) and (3), and that the holes are assumed to be in plane strain for the entirety of the test. Additionally, some variance can be attributed to the hole condition since the holes were drilled and would exhibit a degree of work-hardening at the hole surface.

6. SUMMARY AND CONCLUSIONS

The present work: (i) demonstrated how DIC can be employed to experimentally observe incipient coalescence between drilled holes, (ii) experimentally showed the influence of the intervoid ligament angle on coalescence, and (iii) assessed the ability of Thomason’s cylindrical voids model [4] to predict coalescence in the situation of arbitrary ligament orientation. A suggestion from the work of Scheyvaerts et al. [23] was incorporated into Thomason’s model, significantly improving its predictions.

The literature contains many numerical studies of the coalescence phenomenon but there are far fewer experimental studies on the topic – especially studies in which mathematical coalescence models are quantitatively compared with experimental results, as in the present work.

The principal conclusions of this work are summarized as:

- A digital image history of the experiments was an effective tool for identifying the onset of coalescence. The ability to identify the onset of void coalescence removes the need to assume that IC and ligament failure are separated by a negligibly small strain increment. In addition, DIC enabled calculation of both far-field and near-field (i.e. within the intervoid ligament) strain values.

- Flow stress in the intervoid ligament is greater than the average flow stress because of strain concentration in the ligament prior to incipient coalescence. This calls for use of the ligament flow stress in the PLL equation instead of an average flow stress value (this was recommended by Scheyvaerts et al. [23]). In the present work, the ligament flow stress was obtained using FE simulation results and was taken from an element at the void edge where the stress is greatest (as suggested in [15]).

- An increase in the ligament angle (with respect to the principal loading direction) serves to increase the far-field strain required for IC; however, the misalignment of the ligament with the principal loading direction induces shear stresses that hasten coalescence.

- The Thomason cylindrical voids model gives improved performance in its modified state (Eq. 3), and can be readily incorporated into percolation-type models to provide predictions of void coalescence on the particle-scale. The Thomason model is attractive because of its physical basis and elegance. However, new models accounting for combined tension and shear are a welcome development, enabling better predictions of incipient coalescence in situations of arbitrary ligament orientation. The modified Thomason model showed good qualitative agreement with the experimental results, and percent difference values ranging from 9 to 39% (mean = 25%). Using the modified
Thomason model resulted in reducing the error to less than half of what it was when using the original Thomason model.

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REFERENCES

APPENDIX

After machining, the specimens were cleaned with soap and water and then rubbing-alcohol. Paint primer was used to apply a white coating on the specimens, and a random black-speckle paint layer was applied to the specimen faces to facilitate use of a digital image correlation (DIC) strain measurement system. The DIC software used in the present work was developed by Eberl et al. [24] and was obtained free of charge from the Matlab Central Exchange. The accuracy of the speckle pattern and DIC software was established through comparison tests using a mechanical strain gauge. Recording a digital image history of each experiment was crucial to the present work, enabling both far-field and near-field (i.e. in the intervoid ligament) strain measurement using DIC, and later visual observation and analysis of the coalescence event.