ABSTRACT

In this paper, a 4-PPPS redundantly actuated parallel mechanism and its motion planning method are proposed. The mechanism can be applied to the positioning and alignment of large subassembly in engineering assembly. The optimization of the positioning and alignment trajectory is performed with the minimum energy consumption as the objective function. The system test data shows that the assembly platform and the motion planning method can satisfy the requirements of accuracy, efficiency and stability of the positioning and alignment operations.

Keywords: positioning and alignment, trajectory planning, Euler angles.

RÉSUMÉ

Dans cet article, on propose un mécanisme parallèle redondant actionné à 4-PPPS, ainsi que la méthode de planification de son mouvement. Le mécanisme peut être appliqué au positionnement et l’alignement d’un sous-ensemble plus large dans les procédés d’assemblage en ingénierie. L’optimisation du positionnement et de l’alignement de trajectoire est réalisée en utilisant le minimum de consommation énergétique comme fonction objective. Les données expérimentales du système obtenues démontrent que la plateforme d’assemblage et la planification du mouvement peuvent satisfaire les exigences de précision, d’efficacité et de stabilité des opérations de positionnement et d’alignement.

Mots-clés: positionnement et alignement; planification de trajectoire; angles d’Euler.
1 INTRODUCTION

In the assembly process of large subassemblies, such as aircraft fuselage panels, the precision positioning and alignment is a primary and necessary condition for improving assembly quality [1]. A large number of jigs specially designed according to the analogue value of the assembly part shape and size are employed to align the corresponding assembly part by manual labor in the traditional assembly. The traditional positioning and alignment method makes it difficult to improve efficiency and to decrease cost and lead time because the jigs must be redesigned for the different assembly parts [2]. With the development of flexible assembly technology, serial link robot and Stewart Platform have been widely applied in the positioning and alignment of the assembly part. Serial link robot has a large working volume, low cost, and great flexibility, while its stiffness, accuracy, and repeatability are relatively low. On the contrary, Stewart Platform has higher stiffness, accuracy, and repeatability when compared to serial link robot, but its working volume is much smaller and its cost is higher. Due to these problems, the automated positioning and alignment systems are developed to realize the automated flexible assembly of the large subassemblies, such as the POGO sticks system developed by Brötje-Automation Co. [3], M. Torres Co. [4] and Advanced Integration Technology, Inc. [5], but the corresponding equipment and the automated positioning and alignment method are still under technology blockage. The aim of our research is therefore to break down this blockage.

Giving consideration to flexibility, loading ability, location accuracy and working volume, a 4-PPPS (P denotes a prismatic joint and S denotes a ball joint) redundantly actuated parallel mechanism is proposed for adjusting the assembly part position and orientation; this is shown in Fig. 1. In order to prevent the inner stress of the assembly part when moving to a target position and orientation, every branched chain must run coordinately based on the trajectory calculated beforehand. So, the key problem of the research lies on how to plan the motion trajectory of the assembly part accurately and efficiently.

The polynomial interpolating method [6] which is the traditional trajectory planning method is simple, quick and popularly used nowadays. It usually takes the minimum operating time as the objective function because the improvement of the productivity directly originates from the decrease of the travel operating time [7]. But the method neglected the actuator’s dynamics limitation under the hypotheses that the robot members are treated as completely rigid in the trajectory planning. It yielded discontinuous torques that the real actuators cannot generate and then leads to resonance phenomenon existing in high frequency vibrations. For this reason, many research works on optimal trajectory planning have been carried out. On kinematics optimal trajectory planning aspect for the large subassemblies, Zhang et al. [8] used Euler angle and quintic polynomial to interpolate the intermediate position and orientation between the initial and target position and orientation, and proposed a shortest-time trajectory planning method. Zha et al. [9, 10] regarded the manipulator's trajectory as a ruled surface and interpolated between two positions and orientations by Bezier curves. In order to avoid the singular position during driving parallel mechanism, Ma and Angeles [11] and Merlet [12] conducted independent investigations on the application of quaternion for the position and orientation. Zhu et al. [13] and Yu et al. [14] proposed a 6-degree of freedom (6-DOF) wing trajectory planning method based on quaternion to improve the efficiency and precision of the positioning and alignment of ARJ21 wing. Eldershaw and Cameron [15], Tian and Collins [16], and Yun and Xi [17] used a genetic algorithm to study the trajectory planning problem in joint space, and these methods were based on the polynomial interpolation.

In order to perform a good trajectory planning, the dynamics also need to be taken into account. Dynamic optimal trajectory planning has most methods mainly dwelling on the construction of the objective function. To reduce the stresses and damage to the actuators and the manipulator structure, Hollerbach and Suh [18] took joint torque as the objective function to adjust the coefficients of the joint angles' polynomials and got the minimum joint torque trajectory. Another approach was to take the time integral of the joint torques and acceleration as optimum objective functions [19–21]. The trajectory-
following errors, friction between moving components, dynamic effects on the actuator/transmission and vibration of the structure were respectively decreased or weakened. Although the continuous and stable motion characteristics is necessary, the minimal energy trajectory planning method still has been widely used, such as robots for space or for underwater exploration. Usually the optimal operating time and the minimum energy consumption were considered together as cost functions \([7,20,22]\), and the balance among the operating time, actuator effort and power could be adjusted by choosing the relevant weights.

Based on the above review we can see that there is no method focusing on how to interpolate and optimize motion trajectory of the 4-PPPS parallel mechanism proposed in this paper. Therefore, this paper develops a 6-DOF positioning and alignment method to construct the inverse kinematics and dynamics model for the 4-PPPS parallel mechanism and then discusses its motion trajectory when the positioning and alignment path is uncertain. The optimization of the positioning and alignment time in trajectory planning is divided into two stages: in the first, the minimum positioning and alignment time is searched under the physics limitations of drive unit. This search is in the range of allowed positioning and alignment time via dichotomy increased by degrees. In the second, the optimal positioning and alignment time is globally searched offline between the minimum positioning and alignment time and the maximum allowed one. The minimum energy consumption which is the system’s moving cost is taken as objective function. Furthermore, the effectiveness of the proposed method is verified through computer simulation and then by experimental verification on the experimental prototype, as shown in Fig. 1.

The rest of this paper is organized as follows; Section 2 simply introduces automated positioning and alignment method while sections 3, 4 and 5 build the kinematics, inverse kinematics and dynamics model of 4-PPPS (redundantly actuated parallel mechanism) respectively; Section 6 presents the trajectory planning method using the quintic polynomial approach. Also a searching algorithm of optimal positioning and alignment time aiming at the minimum moving cost is proposed in this section. The theoretical calculation and experimental study are provided in Section 7. Section 8 puts forward the conclusions and points out the future work.

2 AUTOMATED POSITIONING AND ALIGNMENT METHOD

The experimental prototype of 6-DOF large subassembly automated positioning and alignment mechanism is developed in XJTU (Xi’an Jiaotong University, P. R. CHINA) as shown in Fig. 1(a). According to its simplified geometric model in Fig. 1(b), the mechanism can be considered as a 4-PPPS redundantly actuated parallel mechanism with the assembly part equivalent to the moving platform. As shown in Fig. 2, each accurate 3-axis locator capable of moving and locating accurately is composed of \(x, y, \) and \(z\) axes movable sliders which form three prismatic joints driven by high-precision servo motors in the direction of \(x, y,\) and \(z\) axes. A ball-head fixed on the assembly part and a ball-socket fixed on the top of the locator both form the ball joint used in the positioning and alignment of the assembly part.
Fig. 1. 6-DOF large subassembly automated positioning and alignment mechanism: (a) system composition; (b) geometric model.

Fig. 2. Accurate three-axis locator: (a) CAD model; (b) geometric model.

According to the Kutzbach–Grübler formula, the mechanism can control 6 degrees of freedom of the assembly part during the positioning and alignment. The redundantly actuated pattern, namely the x, y, and z axes movable sliders of the four accurate 3-axis locator are respectively driven by the twelve servo motors, can yield the optimal load distribution among the actuators, while the actuator singularity and backlash phenomenon during motion can be avoided and the stresses of the mechanism weakened. The liner feeding motion along every servo driving axis makes the ball-head to rotate freely relative to the ball-socket in all directions to meet the need of the position and orientation adjustment. The flow diagram of the automated positioning and alignment method is shown in Fig. 3. The initial position and orientation of the assembly part can be calculated by the measurement coordinates of the observation points $P_1$, $P_2$, $P_3$ and $P_4$. Based on the initial and the known target position and orientation, the theoretical intermediate position and orientation are confirmed by the trajectory planning. The positioning and alignment error can be obtained by comparing the theoretical position and orientation with the actual position and orientation calculated by the measurement coordinates of the observation points.
3 KINEMATICS MODEL

A generalized coordinate vector $\mathbf{\Omega}$ is defined to describe the position and orientation of the controllable 6 degree of freedom assembly part as shown in Eq. (1):

$$\mathbf{\Omega} = (x, y, z, \phi, \theta, \psi)^T,$$

where the coordinates $x$, $y$ and $z$ are positional displacements which locate the position of the assembly part. The other three coordinates, Euler $z$-$x$-$z$ angles $\phi$, $\theta$ and $\psi$ are used to represent angular displacements which describe the orientation of the assembly part.

In order to describe kinematics of the assembly part, two-coordinate systems are constructed as shown in Fig. 3. The world coordinate frame $O_w$-$x_w$-$y_w$-$z_w$ is fixed to the assembly part and its coordinate axes are not only the same as the coordinate axes of the assembly part design coordinate system but also parallel with the movable direction of the sliders constituting the accurate 3-axis locator. The frame $O_m$-$x_m$-$y_m$-$z_m$ is the measurement coordinate system of the laser tracker. The position of frame $O_m$-$x_m$-$y_m$-$z_m$ is specified with reference to frame $O_w$-$x_w$-$y_w$-$z_w$ by a vector $p_e = (x_e, y_e, z_e)^T$ which defines the coordinates of point $O_m$ with reference to frame $O_w$-$x_w$-$y_w$-$z_w$. The orientation of frame $O_m$-$x_m$-$y_m$-$z_m$ is described with reference to frame $O_w$-$x_w$-$y_w$-$z_w$ by a rotation matrix $^wR_m$, where $r_1$, $r_2$ and $r_3$ are respectively 3x1 unit vectors along the axes of frame $O_m$-$x_m$-$y_m$-$z_m$ and described with reference to frame $O_w$-$x_w$-$y_w$-$z_w$. The rotation matrix $^wR_m$ is given for the used Euler angle representation as:

$$^wR_m = \begin{bmatrix}
-sin\phi sin\psi + cos\phi cos\psi & -cos\phi sin\psi - sin\phi cos\psi & sin\theta sin\phi \\
-cos\phi sin\psi + sin\phi cos\psi & -sin\phi sin\psi + cos\phi cos\psi & -sin\theta cos\phi \\
sin\theta sin\psi & sin\theta cos\psi & cos\theta
\end{bmatrix}.$$
Before proceeding to the inverse kinematic problem, it is needful to express the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the assembly part with reference to frame $O_{w}-x_{w}y_{w}z_{w}$ as functions of the first and second time derivatives of the Euler angles $(\phi, \theta, \psi)$ and $(\dot{\phi}, \dot{\theta}, \dot{\psi})$.

$$
\boldsymbol{\omega} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} = R(\phi, \theta, \psi) \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \cos \phi & \sin \phi \cos \theta \\ -\cos \phi & \sin \phi & \cos \phi \sin \theta \\ 1 & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}.
$$

(3)

The angular acceleration of the assembly part is obtained by differentiating Eq. (3):

$$
\boldsymbol{\alpha} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \begin{bmatrix} 0 & \cos \phi & \sin \phi \sin \theta \\ -\cos \phi & \sin \phi & -\cos \phi \sin \theta \\ 1 & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\phi} \sin \phi & \phi \cos \phi \cos \theta + \dot{\theta} \sin \phi \cos \theta \\ -\dot{\phi} \sin \phi & \phi \cos \phi & \phi \sin \phi \sin \theta - \dot{\theta} \cos \phi \cos \theta \\ 0 & 0 & -\dot{\phi} \sin \theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}.
$$

(4)

4 INVERSE KINEMATICS MODEL

The inverse kinematic problem of the assembly part is concerned with the determination of the feeding displacements of the twelve servo motors and their time derivatives corresponding to a given Cartesian position and orientation of the assembly part in terms of three positional displacements, and three Euler angular displacements and their time derivatives. So, the solutions for the inverse position, velocity, acceleration and jerk kinematics are presented as follows:

4.1 Inverse Position Kinematics

Referring back to the Figure 3, the measurement coordinate vector of the observation point $P_j$ is expressed as $\mathbf{p}_j^m = (x_j^m, y_j^m, z_j^m)^T$, and the corresponding coordinate vector in frame $O_{w}-x_{w}y_{w}z_{w}$ is expressed as $\mathbf{p}_j^w = (x_j^w, y_j^w, z_j^w)^T$. The relationship between $\mathbf{p}_j^m$ and $\mathbf{p}_j^w$ is as follows:

$$
\mathbf{p}_j^w = R_{w} \mathbf{p}_j^m + \mathbf{p}_e.
$$

(5)

where $j=1,2,3,4$. According to Eq. (5), it is obvious that the initial position and orientation vector $\boldsymbol{\Omega}_i$ of the assembly part composed of the six unknown independent parameters can be solved when the number of the observation point is not smaller than two. Similar to the calibration of $\boldsymbol{\Omega}_t$, the target position and orientation vector $\boldsymbol{\Omega}_n$ of the assembly part is usually given as the known technology condition or can be obtained through measuring the observation points on the reference assembly part. So the position and orientation requirement $\Delta \boldsymbol{\Omega}$ of the assembly part can be formulated as:

$$
\Delta \boldsymbol{\Omega} = \boldsymbol{\Omega}_n - \boldsymbol{\Omega}_t.
$$

(6)

The points $P_1$, $P_2$, $P_3$ and $P_4$ are used to calculate the vector $\Delta \boldsymbol{\Omega}$, but the corresponding variety of their coordinates in the frame $O_{w}-x_{w}y_{w}z_{w}$ cannot be directly utilized to control servo motors because the relation between the observation points and driving points is not unknown. Since the motions of the ball joints are driven directly by the motors, the ball joint centers $Q_1$, $Q_2$, $Q_3$ and $Q_4$ are defined as the driving points. The measurement coordinate vector of the ball joint center $Q_i$ is expressed as
\[
\mathbf{q}_j^m = (x_j^m, y_j^m, z_j^m)^T, \quad \text{and the corresponding coordinate vector in frame } O_w-x_wy_wz_w \text{ is expressed as }
\mathbf{q}_j^w = (x_j^w, y_j^w, z_j^w)^T. \quad \text{The relationship between } \mathbf{q}_j^m \text{ and } \mathbf{q}_j^w \text{ is as follows:}
\]
\[
\mathbf{q}_j^w = m_R \cdot \mathbf{q}_j^m + \mathbf{q}_e, \tag{7}
\]
where \( \mathbf{q}_e \) is the position vector of the assembly part when adjusting the position and orientation.

### 4.2 Inverse Velocity Kinematics

The velocity vectors of the ball joints are obtained by differentiating Eq. (7) with respect to time:

\[
\dot{\mathbf{q}}_j^w = m \dot{R}_w \cdot \mathbf{q}_j^m + \dot{\mathbf{q}}_e = \mathbf{J}_j^{-1} \left( \begin{array}{c} \dot{\mathbf{q}}_e \\ \mathbf{0} \end{array} \right), \tag{8}
\]

where

\[
\mathbf{J}_j^{-1} = \begin{pmatrix} I_{3 \times 3} & (m_R \cdot \mathbf{q}_j^m) \times (1 \ 0 \ 0)^T \\ m_R \cdot \mathbf{q}_j^m \times (0 \ 1 \ 0) \\ m_R \cdot \mathbf{q}_j^m \times (0 \ 0 \ 1) \end{pmatrix} \tag{9}
\]

Now substituting Eq. (3) into Eq. (8) yields

\[
\dot{\mathbf{q}}_j^w = \mathbf{J}_j^{-1} \mathbf{J}^1 \dot{\mathbf{Q}} = \mathbf{J}^{-1} \dot{\mathbf{Q}}, \tag{10}
\]

where

\[
\mathbf{J}^{-1} = \mathbf{J}_1^{-1} \mathbf{J}_2^{-1} \text{ is the inverse Jacobian matrix of the 4-PPPS parallel mechanism.}
\]

The mechanism is in a singular position when \( \det(\mathbf{J}^{-1}) = 0 \). Such condition will occur when either \( \det(\mathbf{J}_1^{-1}) = 0 \) or \( \det(\mathbf{J}_2^{-1}) = 0 \). These two types of singularity are the configuration and formulation singularities respectively, as noted by Ma and Angeles [11].

For the conditions \( \det(\mathbf{J}^{-1}) = 0 \), it is difficult to find analytically since an analytical expression for the determinant of \( \mathbf{J}_1^{-1} \) is not available, but the assembly part cannot be adjusted to the singularity position and orientation proposed in Refs. [11] and [12] throughout the whole workspace. Hence, such a singularity is avoided under the finite rotation premise of the assembly part from the initial to the target position and orientation.
For the conditions \( \det(J_2^{-1}) = 0 \), it is associated with the \( z-x-z \) Euler angle formulation used and it will occur when \( \theta=0, \pm \pi, \ldots, \pm n\pi \). For the coordinate system assignment as shown in Fig. 4(a), the singularity will occur for all horizontal positions of the platform. Since this situation cannot be allowed in practice, singularity can be avoided either by changing the Euler angle formulation or by changing the coordinate system assignment. The latter solution is used, as shown in Fig. 4(b). The formulation singularity occurs when the platform is in a vertical position. This is far removed from normal operational configurations of the above parallel mechanism and will not be encountered in this application.

4.3 Inverse Acceleration and Jerk Kinematics

The acceleration vectors of ball joints are obtained by differentiating Eq. (10) with respect to time.

\[
\ddot{q}_j^w = J_1^{-1} J_2^{-1} \ddot{\Omega} + \frac{dJ_1^{-1}}{dt} J_2^{-1} \dot{\Omega} + J_1^{-1} \frac{dJ_2^{-1}}{dt} \dot{\Omega}.
\]

The jerk vectors of ball joints are obtained by differentiating Eq. (12) with respect to time.

\[
\dddot{q}_j^w = J_1^{-1} J_2^{-1} \dddot{\Omega} + 2 \left( \frac{dJ_1^{-1}}{dt} J_2^{-1} + J_1^{-1} \frac{dJ_2^{-1}}{dt} \right) \ddot{\Omega} + \left( \frac{d^2J_1^{-1}}{dt^2} J_2^{-1} + 2 \frac{dJ_1^{-1}}{dt} \frac{dJ_2^{-1}}{dt} + J_1^{-1} \frac{d^2J_2^{-1}}{dt^2} \right) \dot{\Omega}.
\]

5 DYNAMICS MODEL

5.1 Dynamics Model of the Assembly Part

According to the Newton-Euler method, dynamics model of the assembly part can be formulated as:

1) Dynamics equations during adjusting the position

\[
\sum_{j=1}^{d} m_{R_j} \ddot{F}_j^r = m \ddot{p}_e - mG,
\]
where $F_j$ is the interaction force vector between the assembly part and $j$th 3-axis locator with reference to frame $O_{w-x_wy_wz_w}$, $m$ is the mass of the assembly part, $I$ is the 3×3 identity matrix, $G = [0 \ 0 \ g]^T$ ($g$ is gravitational acceleration).

2) Dynamics equations during adjusting the orientation

$$\sum_{j=1}^{4} \hat{q}_j^w F_j' = I_s \alpha + \bar{\alpha} I_s \bar{\alpha},$$

(15)

where $\hat{q}_j$ is anti-skew-symmetric matrix of the position vector $q_j^w$, $\bar{\alpha}$ is anti-skew-symmetric matrix of angular velocity $\alpha$, $I_s$ is the moment of inertia matrix of the assembly part with reference to frame $O_{w-x_wy_wz_w}$.

3) Solution of the interaction force

Combining Eq. (14) and (15), dynamics model of the assembly part is constructed as follows:

$$\mathbf{P} = \mathbf{WF}' ,$$

(16)

where

$$F' = \begin{pmatrix} F_1' & F_2' & F_3' & F_4' \end{pmatrix}^T ,$$

(17)

$$P = \begin{pmatrix} m\ddot{q}_w \ -mG \\ I_s \alpha + \bar{\alpha} I_s \bar{\alpha} \end{pmatrix} ,$$

(18)

$$W = \begin{pmatrix} mR_w & mR_w & mR_w & mR_w \\ \bar{q}_j^w & \bar{q}_j^w & \bar{q}_j^w & \bar{q}_j^w \end{pmatrix} .$$

(19)

According to the distribution of the accurate 3-axis locator and the servo motors shown in Fig. 1, the interaction force vector $F'$ has 12 elements in Eq. (17) and the number of equations is only six. Solving Eq. (17) by Moore-Penrose pseudo-inverse matrix, the general form of solution is as follows:

$$F_j' = W^+ P + (I_{12} - W^+ W) \varepsilon ,$$

(20)

where $W^+ = (W^T W)^{-1} W^T$ is a generalized inverse matrix of $W$, $I_{12}$ is the 12×12 identity matrix; $\varepsilon$ is an arbitrary selected 12×1 vector which decides that $F_j'$ has multiple solutions.

Based on the attribute that the low driving force improves system control precision and moving stability efficiently with ease, selecting $\varepsilon = 0$, and the minimum 2-norm solution can be calculated as follows:

$$F_j' \| = W^+ P .$$

(21)
5.2 Dynamics Model of the Single Accurate 3-axis Locator

The driving force vector of the \(j\text{th}\) 3-axis locator is expressed as \(F_j = (F_{jx}, F_{jy}, F_{jz})^T\). \(F_{jx}, F_{jy},\) and \(F_{jz}\) are the driving forces of the \(j\text{th}\) 3-axis locator in the direction of \(x, y,\) and \(z\) axes respectively. The dynamics equations of the single accurate 3-axis locator can be formulated as:

\[
F_j = H_j \ddot{q}_j^w + m J \dot{q}_j^w - m J \dot{q}_j^w - m J G + f_j, \tag{22}
\]

where \(H_j = \text{diag}\left( m_{jx} + m_{jy} + m_{jz}, m_{jy} + m_{jz}, m_{jz} \right)\) is a diagonal matrix of the moving component mass. \(m_{jx}, m_{jy},\) and \(m_{jz}\) are the mass of the movable sliders of the \(j\text{th}\) 3-axis locator in the direction of \(x, y,\) and \(z\) axes respectively. \(f_j\) is the frictional forces vector of the \(j\text{th}\) 3-axis locator expressed as:

\[
f_j = \begin{pmatrix} f_{jx} \\ f_{jy} \\ f_{jz} \end{pmatrix} = \Xi \Theta \begin{pmatrix} m_{jx} \dot{q}_{jx}^w \\ m_{jy} \dot{q}_{jy}^w \\ 0 \end{pmatrix} + \begin{pmatrix} (m_{jx} + m_{jy} + m_{jz})g \\ (m_{jy} + m_{jz})g \\ 0 \end{pmatrix} + \xi m J \dot{q}_j^w , \tag{23}
\]

where \(f_{jx}, f_{jy},\) and \(f_{jz}\) are the frictional forces of the \(j\text{th}\) 3-axis locator in the direction \(x, y,\) and \(z\) axes respectively. \(\ddot{q}_j^w\) is the acceleration of the \(j\text{th}\) 3-axis locator in the \(z\) direction.

\[
\xi = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \tag{24}
\]

\[
\Xi = \text{diag}\left( \text{sign}\left( \dot{q}_{jx}^w \right), \text{sign}\left( \dot{q}_{jy}^w \right), 0 \right)^T, \tag{25}
\]

\[
\Theta = \text{diag}\left( \mu, \mu, 0 \right)^T, \tag{26}
\]

where \(\dot{q}_{jx}^w\) and \(\dot{q}_{jy}^w\) are the accelerations of the \(j\text{th}\) 3-axis locator in the direction of \(x\) and \(y\) axes respectively. \(\mu\) is friction coefficient.

By selecting the lead screw as the equivalent component of the \(j\text{th}\) 3-axis locator, the equivalent mechanical model can be formulated as:

\[
F_j v' = (\boldsymbol{\tau}_j - T_j) \omega', \tag{27}
\]

where \(v'\) is the translatory velocity along the driving direction of the slider, \(\omega'\) is angular velocity of the motor. \(\boldsymbol{\tau}_j = (\tau_{jx}, \tau_{jy}, \tau_{jz})^T\) is the actuating torque vector of the \(j\text{th}\) 3-axis locator. \(T_j\) is the loading torque vector expressed as:

\[
T_j = \frac{2\pi}{l} \text{diag}\left( J_{s1} + J_{m1}, J_{s2} + J_{m2}, J_{s3} + J_{m3} \right) \ddot{q}_j^w, \tag{28}
\]
where \( l \) is the lead screws. \( J_S \) and \( J_M \) are the equivalent inertia of the screws and motors respectively. Substituting Eq. (21–26) and (28) into Eq. (27) yields:

\[
\tau_j = IF_j + T_j,
\]

where \( F = \text{diag}(l/2\pi, l/2\pi, l/2\pi)^T \).

6 TRAJECTORY PLANNING

The trajectory planning is to solve intermediate position and orientation between the initial and the target position and orientation during the automated positioning and alignment, and then to plan the motion trajectory correctly under the premise of ensuring the assembly precision and the assembly efficiency. Hence, based on the kinematics model constructed with Eq. (7–13), to solve theoretical motion trajectory of the assembly part, it is essential to consider the following: a) constraints; b) trajectory representation; c) optimization objective; d) optimization algorithm.

6.1 Constraints

In engineering application, an assembly part should move smoothly and efficiently. The beginning and terminating velocities as well as accelerations should be zero to eliminate impact. So, a large number of constraints must be met during the automated positioning and alignment, such as kinematics boundary conditions and physical constraints as described below:

1. Kinematics Constraints

   Position and orientation:
   \[
   \omega_j = \omega(t = 0) \quad \omega_n = \omega(t = T_{\text{opt}}).
   \]

   Velocity:
   \[
   \dot{\omega}_j = \dot{\omega}(t = 0) = 0 \quad \dot{\omega}_n = \dot{\omega}(t = T_{\text{opt}}) = 0.
   \]

   Acceleration:
   \[
   \ddot{\omega}_j = \ddot{\omega}(t = 0) = 0 \quad \ddot{\omega}_n = \ddot{\omega}(t = T_{\text{opt}}) = 0.
   \]

   where \( t \) is the positioning and alignment time variable, \( T_{\text{opt}} \) is the optimal positioning and alignment time.

2. Physical Constraints

   In order to avoid resonance of mechanism in engineering practice, the displacements, velocities, accelerations, jerks of the assembly part and the actuating torque must satisfy the following constraints:

   Displacements:
   \[
   [q_j^w]_{\text{min}} \leq q_j^w(t) \leq [q_j^w]_{\text{max}}.
   \]

   Velocities:
   \[
   [\dot{q}_j^w]_{\text{min}} \leq \dot{q}_j^w(t) \leq [\dot{q}_j^w]_{\text{max}}.
   \]

   Accelerations:
   \[
   [\ddot{q}_j^w]_{\text{min}} \leq \ddot{q}_j^w(t) \leq [\ddot{q}_j^w]_{\text{max}}.
   \]

   Jerks:
   \[
   [\dddot{q}_j^w]_{\text{min}} \leq \dddot{q}_j^w(t) \leq [\dddot{q}_j^w]_{\text{max}}.
   \]

   Actuating Torque:
   \[
   [\tau_j]_{\text{min}} \leq \tau_j(t) \leq [\tau_j]_{\text{max}}.
   \]
6.2 Trajectory Representation

The above analysis reveals that the six kinematic constraints must be met during the automated positioning and alignment. Therefore, the motion trajectory of the assembly part should be fitted by quintic polynomial, parameterized with the positioning and alignment time. The trajectory function can be formulated as:

\[
\begin{bmatrix}
\Omega(t) \\
\dot{\Omega}(t) \\
\ddot{\Omega}(t)
\end{bmatrix} =
\begin{bmatrix}
t^5 & t^4 & t^3 & t^2 & t & 1 \\
5t^4 & 4t^3 & 3t^2 & 2t & 1 & 0 \\
20t^3 & 12t^2 & 6t & 2 & 0 & 0
\end{bmatrix} \begin{bmatrix}
a_5 \\
a_4 \\
a_3 \\
a_2 \\
a_1 \\
a_0
\end{bmatrix},
\]

(38)

where \(a_0, a_1, \ldots, a_5\) are unknown parameters.

Substituting Eq. (30–32) into Eq. (38), the unknown variables \(a_i\) \((i=1,2,3,4,5)\) can be solved as shown in Eq. (39):

\[
\begin{bmatrix}
a_5 \\
a_4 \\
a_3 \\
a_2 \\
a_1 \\
a_0
\end{bmatrix} =
\begin{bmatrix}
\frac{6\Delta\Omega}{T_{\text{op}}^5} \\
\frac{-15\Delta\Omega}{T_{\text{op}}^4} \\
\frac{10\Delta\Omega}{T_{\text{op}}^3} \\
0 \\
0 \\
\Omega_i
\end{bmatrix}.
\]

6.3 Objective Optimization Function

The motion quality depends strongly on the expression adopted for the objective function \(E\). In fact, \(E\) represents the moving cost of the automated positioning and alignment process. Hence, it would be judicious to include significant physical parameters related directly to the global behavior of the mechanism and the productivity of the industrial process. The following general expression, Eq. (40) and (41), is a balance between the positioning and alignment time \(t\) and moving cost \(E\). The minimum posture alignment time \(T_s\) which meets all the constraint conditions is first calculated using Eq. (40) before proceeding to optimize the system moving cost \(E\) using Eq. (41) with consideration to assembling efficiency.

\[
\min_{T_s \in [T_{\text{min}} \ T_{\text{max}}]} t = \int_0^{T_s} dt,
\]

(40)

\[
\min_{E} E = \sum_{j} \int_{T_s}^{T_{\text{max}}} \left| \dot{q}_j^* F_j \right| dt,
\]

(41)

where \([T_{\text{min}} \ T_{\text{max}}]\) is the allowed positioning and alignment time range of the assembly part. \(T_s\) is the minimum positioning and alignment time meeting all the constraint conditions. \(F_j\) is the actuating force vector of the corresponding driving motor.
6.4 Optimization Method

The method proposed in this paper optimizes the positioning and alignment time $t$ and system moving cost $E$ with Eq. (40) and (41) as optimal objectives and achieves the searching for the optimal positioning and alignment time $T_{\text{opt}}$ corresponding to the optimization moving cost. The different steps of the algorithmic implementation are as follows:

**Step 1. User data acquisition**
- Measure the initial position and orientation $\Omega_i$ of the assembly part
- Read the target position and orientation $\Omega_n$ of the assembly part
- Set the sampling time interval $\Delta t$, calculation precision $\delta$ and the range of the allowed positioning and alignment time $[T_{\text{min}}, T_{\text{max}}]$

**Step 2. Inverse Kinematics and Dynamics Model:**
- Construct the system inverse kinematics equations, Output: $\dot{q}_i^*(t), \ddot{q}_i^*(t), \dot{q}_j^*(t)$
- Construct the system dynamics equations, Output: $\tau_j(t)$
- Construct kinematics and physical constraint

**Step 3. Search for the shortest positioning and alignment time $T_s$ in the range $[T_{\text{min}}, T_{\text{max}}]$.
- $t = T_{\text{min}}$. If $q_i^*(t), \dot{q}_i^*(t), \ddot{q}_i^*(t)$ and $\tau_j(t)$ meet the Eq. (33–37), meaning that $T_s = T_{\text{min}}$; else, set $t = T_{\text{max}}$, judge whether $q_i^*(t), \dot{q}_i^*(t), \ddot{q}_i^*(t)$ and $\tau_j(t)$ meet the Eq. (33–37). If so, $T_{\text{mid}} = (T_{\text{min}} + T_{\text{max}})/2$; else, $[T_{\text{min}}, T_{\text{max}}]$ is unreasonable, go back to step 1.
- $t = T_{\text{mid}}$. If $q_i^*(t), \dot{q}_i^*(t), \ddot{q}_i^*(t)$ and $\tau_j(t)$ meet the Eq. (33–37), $T_{\text{max}} = T_{\text{mid}}$; else, $T_{\text{min}} = T_{\text{mid}}$.
- If $T_{\text{max}} - T_{\text{mid}} \leq \delta$, $T_s = T_{\text{mid}}$; else, go back to step 2.

**Step 4. Searching for the optimal posture alignment time $T_{\text{opt}}$**
- Search $T_{\text{opt}}$ by Eq. (41) with consideration to assembling efficiency in the range $[T_s, T_{\text{max}}]$

**Step 5. Calculating Control Variables**
- $t = T_{\text{opt}}$, $\Delta t = 0.1s$, calculate control variables: $q_i^*(t), \dot{q}_i^*(t), \ddot{q}_i^*(t)$ and $\tau_j(t)$.

The whole flowchart of the algorithm is shown as Fig. 5:
Fig. 5. Flowchart of trajectory optimization algorithm.
7 SYSTEM TEST

In this section the theoretical calculation and experimental study are carried out to verify the feasibility of the proposed motion planning method for the 4-PPPS parallel mechanism developed in XJTU.

The measurement coordinate vectors of the observation points \( P_1, P_2, P_3 \) and \( P_4 \) are \((-1490.1279, 3473.9877, 445.2134)\), \((-490.8155, 3510.959, 405.3513)\), \((-509.088, 3526.0402, -45.2122)\) and \((-1509.1911, 3498.8254, -5.5643)\), the corresponding coordinate vectors in frame \( O_{w-x_w y_w z_w} \) are \((225.895, 0, 500.003)\), \((225.323, 0, -500.787)\), \((-225.863, 9.765, 500.424)\). The ball joints center \( Q_1, Q_2, Q_3 \) and \( Q_4 \) are \((165.915, -320.025, 500.003)\), \((165.319, -319.993, -500.787)\), \((-285.857, -319.989, -500.787)\) and \((-285.841, -310.221, 500.424)\). The units of the measurement coordinate vectors of points \( P_j \) and \( Q_j \) is \( \text{mm} \). The position and orientation requirement \( \Delta \Omega \) between the initial and the target position and orientation of the assembly part is \((-10 \text{mm}, -15 \text{mm}, 22 \text{mm}, -0.015 \text{rad}, 0.02 \text{rad}, -0.025 \text{rad})\). The mass of the assembly part equals 500kg and its inertia matrix is \( \text{diag} \) \((780, 950, 830)\) in frame \( O_{w-x_w y_w z_w} \). Parameters of the accurate 3-axis locator are given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the x-axis slider ( m_{x} ) (kg)</td>
<td>30</td>
</tr>
<tr>
<td>Mass of the y-axis slider ( m_{y} ) (kg)</td>
<td>30</td>
</tr>
<tr>
<td>Mass of the z-axis slider ( m_{z} ) (kg)</td>
<td>80</td>
</tr>
<tr>
<td>Lead of the screws ( l ) (mm)</td>
<td>5</td>
</tr>
<tr>
<td>Friction coefficient ( \mu )</td>
<td>0.02</td>
</tr>
<tr>
<td>Equivalent inertia of the x-axis and y-axis screws ( J_{xy} ) (kg·m²)</td>
<td>( 5.5 \times 10^{-5} )</td>
</tr>
<tr>
<td>Equivalent inertia of the z-axis screws ( J_{xz} ) (kg·m²)</td>
<td>( 10 \times 10^{-5} )</td>
</tr>
<tr>
<td>Equivalent inertia of the x-axis and y-axis motors ( J_{xy} ) (kg·m²)</td>
<td>( 1.51 \times 10^{-4} )</td>
</tr>
<tr>
<td>Equivalent inertia of the z-axis motor ( J_{z} ) (kg·m²)</td>
<td>( 13.7 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

7.1 Theoretical Calculation

Theoretical calculation is carried out in MATLAB®. The allowed positioning and alignment time range is \([0, 30]\) and the sampling time interval \( \Delta t \) and calculation precision \( \delta \) are all set at 0.1s. The maximum displacements, velocities, accelerations, jerks and actuator forces along every servo driving axis are 35mm, 3mm/s, 0.5mm/s², \( 5 \times 10^{-3} \)mm/s³ and 2kN respectively. Figure 6 shows the searching process of the minimum positioning and alignment time \( T_s \). Figure 7 shows the optimum positioning and alignment time \( T_{opt} \) is 18.4s. The displacements, velocities, accelerations, jerks trajectories corresponding to \( T_{opt} \) of the ball joint center \( Q_1 \) and the actuating force trajectories of the motor are obtained, as shown in Figs. 8–12, which can be used for the motor control program. The corresponding actuating torque trajectories and the interaction force trajectories between the assembly part and the locator are shown in Figs. 13 and 14. The results corresponding to the ball joint centers \( Q_2, Q_3 \) and \( Q_4 \) are similar.
Fig. 6. Iterative searching $T_s$.

Fig. 7. Global searching $T_{opt}$.

Fig. 8. Displacement trajectories of ball joint center $Q_1$. 

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Fig. 9. Velocity trajectories of ball joint center $Q_1$.

Fig. 10. Acceleration trajectories of ball joint center $Q_1$.

Fig. 11. Jerk trajectories of ball joint center $Q_1$. 
Fig. 12. Actuating force trajectories of motor.

Fig. 13. Actuating torque trajectories of motor.

Fig. 14. Interaction force trajectories between assembly part and locators.
The results above show that the moving cost corresponding to $T_s$ (15.8s) is not optimal, and the positioning and alignment time corresponding to minimum moving cost is 18.4s, as shown in Figs. 6 and 7. From Figures 8–14, it can be observed that the motor has good dynamic characteristics and steady response performances. It expresses that the position and orientation variations of the assembly part are slight and smooth and that the maximum displacement, velocities, accelerations, jerk and actuating force meet the engineering constraints as well as the design of accurate 3-axis locator. The coordinate system assignments and the finite rotation premise avoid effectively the configuration singularity and formulation singularity of the 4-PPPS parallel mechanism proposed in this study. Therefore the searching algorithm of the optimal positioning and alignment time and the motion planning method are feasible and effective. Furthermore, Figures 12–14 indicate that the interaction forces, actuating forces and actuating torque in the z direction are much larger than the ones in the x and y directions due to the large mass of the assembly part; the assembly part section connected with the locator through the ball joint must have enough stiffness for reducing deformation.

7.2 Experimental Study
From the experiment on the positioning and alignment of assembly part using the 4-PPPS parallel mechanism developed in XJTU as shown in Fig. 1(a), the comparison of the measured displacement trajectories and the theoretical ones is carried out and the motion planning method proposed in this paper is further verified from looking further into the following two aspects:

- **Positioning and alignment process**

  The measured displacement trajectories of the observation points $P_j$ cannot be compared with the theoretical ones of the ball joint center $Q_j$ directly, but the measured displacement data of the ball joint center $Q_j$ can be solved as follows as shown in Eq. (42) below:

  \[ \overrightarrow{O_xQ_j} = \overrightarrow{O_wP_j} + \overrightarrow{P_jQ_j} \]  

  \[ \text{(42)} \]

  where $\overrightarrow{P_jQ_j}$ is a vector which is drawn from point $P_j$ to $Q_j$ in frame $O_w-x_wy_wz_w$.

  Furthermore, the measured data transformed via Eq. (42) cannot still be compared with the theoretical ones of the ball joint center $Q_j$ because the measured period of the laser tracker is not synchronized with the controlling clock frequency of the 4-PPPS parallel mechanism. So, in order to weaken the asynchronism of the corresponding data, the new data points are sampled with the controlling clock frequency of the 4-PPPS parallel mechanism on the fitted curves after the measured data transformed via Eq. (42) is first fitted to the curve with quintic polynomial. The measured displacement trajectories of the ball joint center $Q_j$ from the initial to the target position and orientation is reconstructed through these new sampling data points.

  Taking the ball joint center $Q_1$ as an example, the difference between the theoretical and the measured displacement trajectories are shown in Fig. 15. The curve $C_{Q_1}^{\text{m}}$ and $C_{Q_1}^{\text{t}}$ is the measured and theoretical displacement trajectory of the ball joint center, respectively. The corresponding results of the ball joint centers $Q_2$, $Q_3$ and $Q_4$ are similar. The similarity expressed by the difference between the theoretical trajectory curves and the measured ones can be solved by calculating the Pearson correlation coefficient. The correlation coefficients are listed in Table 2. The correlation coefficients corresponding to the displacement trajectories of all the ball joint centers are more than 0.9990, meaning that the measured and theoretical displacement trajectories are highly relevant, namely that the practical displacement trajectories are fitted properly with the theoretical ones and the motion planning method is completely feasible during automated positioning and alignment of the assembly part.
Fig. 15. Comparison of the measured and theoretical displacement trajectory of ball joint center $Q_1$.

Table 2. Correlation coefficients of displacement trajectory.

<table>
<thead>
<tr>
<th>theoretical trajectory</th>
<th>measured trajectory</th>
<th>Correlation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\bar{\Theta}}^t$</td>
<td>$C_{\bar{\Theta}}^m$</td>
<td>x</td>
</tr>
<tr>
<td>$C_{\bar{\Theta}}^s$</td>
<td>$C_{\bar{\Theta}}^m$</td>
<td>0.9996</td>
</tr>
<tr>
<td>$C_{\bar{\Theta}}^c$</td>
<td>$C_{\bar{\Theta}}^m$</td>
<td>0.9997</td>
</tr>
<tr>
<td>$C_{\bar{\Theta}}^r$</td>
<td>$C_{\bar{\Theta}}^m$</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

● Final localization accuracy

Figures 16 and 17 show the state of the assembly part before and after the positioning and alignment, and the precision positioning of the connecting hole after the positioning and alignment of the assembly part. Positioning accuracy of the observation points $Q_i$ are listed in Table 3. The results show that the assembly quality in this study achieved a $\pm 0.4$ mm positioning accuracy in the direction of $x$ and $z$ axes, while positioning accuracy in the direction of $y$ axis is lower than $\pm 1$ mm. The radical reasons for the accuracy distribution are that the $y$ axis is designed as a cantilever structure and the cantilever beams connecting the ball joints produce the deflection with a bending moment under the gravity of the assembly part.

Table 3. Positioning accuracy of the ball joint centers.

<table>
<thead>
<tr>
<th>Ball joint centers</th>
<th>Position accuracy (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>0.042</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>-0.162</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>-0.211</td>
</tr>
<tr>
<td>$Q_4$</td>
<td>-0.305</td>
</tr>
</tbody>
</table>
8 CONCLUSIONS

A general assembly platform, 4-PPPS Redundantly Actuated Parallel Mechanism is developed for the automated flexible assembly of large subassemblies such as fuselage panel. The parallel mechanism is composed of four accurate 3-axis locators, laser tracker and assembly part automated positioning and alignment software, and successfully achieves the required automated positioning and alignment of the assembly part.

The positioning and alignment method can be used not only for normal assembly part but also for other large subassemblies, and the method provides theoretical and practical basis for the assembly part automation connection of China-made large aircraft. Furthermore, this research will continue on improving positioning accuracy by:

- Strengthening the stiffness of cantilever beam.
- Compensating the feeding motion of y-axis.
- Calibrating the driving orientation of every accurate 3-axis locators with reference to frame \( O_w-x_wy_wz_w \) accurately.

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REFERENCES


