ABSTRACT
We experimentally studied natural convection processes inside terracotta flues as a part of a larger numerical study of ancient Roman baths. The air, heated in a plenum below the wall, rose through the tubes. Two clusters of thermocouples, equally spaced in the flues, measured temperatures throughout the thickness of the wall. The data from the two clusters proved to be measurably different. The resulting convective heat transfer coefficients determined using the bottom cluster, showed no dependence on the plenum temperature. The measured convective heat transfer coefficient was between 6.2 and 7.6 W/m$^2$·°C, with an average of 7.0 W/m$^2$·°C.

Keywords: convective heat transfer coefficient; natural convection; Roman baths.

DÉTERMINATION DU COEFFICIENT D'ÉCHANGE D'AIR CHAUD PAR CONVECTION À TRAVERS DES CONDUITS EN TERRE CUITE

RÉSUMÉ
Nous avons étudié, à des fins expérimentales, dans le cadre d’une étude élargie sur les anciens bains romains, le processus de convection naturelle à l’intérieur des conduits en terre cuite. L’air chauffé dans un plénum, situé au bas du mur, s’élevait à travers des tubes. Deux réseaux de thermocouples, disposés à égale distance dans les conduits, évaluaient la température à travers l’épaisseur du mur. Les données des deux réseaux montrèrent des différences mesurables. Les résultats des coefficients d’échange d’air chaud par convection, réalisés en utilisant le thermocouple situé en bas du mur, ne démontraient aucune dépendance sur la température du plénum. Le coefficient d’échange de chaleur par convection mesuré se situait entre 6.2 et 7.6 W/m$^2$·°C, avec une moyenne de 7.0 W/m$^2$·°C.

Mots-clés : coefficient d’échange d’air chaud par convection ; convection naturelle ; bains romains.
1. INTRODUCTION

The standard method for heating, ventilation, and air conditioning (HVAC) systems is using mixing air to manage temperature, air quality, etc. More and more systems, however, are combining this with radiant floor heating and cooling. While this interpretation is relatively new, the idea of using a hot fluid beneath a raised floor to heat the room above was invented by the Greeks, and subsequently developed by the Romans, to heat bathing complexes (Fig. 1a). They took it one step further by including wall heating. Hot exhaust gases from fires below would rise up terracotta pipes, called *tubuli*, inlaid in the wall. We are using computational fluid dynamics (CFD) to investigate the largely unknown thermal environment in the *caldarium*, or hot bath room, in the Baths of Caracalla in Rome [1], using the *caldarium* of a building built for the NOVA television series [2] as a validation model (Fig. 1b). However, as the main boundary conditions of the CFD model and the driving force behind the air circulation, an accurate assessment of the heating capabilities of these inlaid flues is necessary. This paper describes an experiment to investigate how much heat was transferred from the exhaust to the wall (Fig. 2). The results could have an impact on future HVAC radiant heating systems. Without fans, the Roman system was able to reach high room temperatures. With modern adaptations, this system could provide a new, more energy efficient way to heat and cool buildings. First, however, the fundamentals must be understood, namely the convective heat transfer coefficient (CHTC) of the air rising through the tubes.

Many of the studies on CHTCs involve a warm surface heating cool air and seek to determine how much heat is drawn away from the surface. Of these, few investigate scenarios comparable to the situation presented here, including analyses of enclosures, pipes, and ducts. Most of these proposed relationships between the Rayleigh (*Ra*; Eq. 4) and Nusselt (*Nu*; Eq. 3) numbers which will be shown later to be inappropriate for this experiment. However, it is important to note the ranges of Nusselt numbers for comparative purposes. One of the more interesting is the study by Hatami and Bahadorinejad of a vertical flat-plate solar...
Fig. 1. (a) Diagram of Roman heating system showing key features. Inset is of a terracotta flue used as a tubulus substitute. (b) A CAD model of the replica baths built for NOVA showing the various rooms including the caldarium.

Fig. 2. (a) Picture of experimental setup. (b) Picture looking down the rightmost flue showing the layers of the wall (Photos by Matt Oetelaar).
air heater [3]. Nusselt numbers ranged from $2.9 \times 10^2$ to $3.7 \times 10^2$ ($1.2 \times 10^{10} < Ra < 3.0 \times 10^{10}$) for channel flow and 1.4 to 1.9 ($3.0 \times 10^4 < Ra < 1.2 \times 10^5$) for enclosures. This was very close to those for the Roman heating system except the surfaces are heating the air instead of the other way around. Fossa et al., used a similar setup to examine the possibility of convection to cool photovoltaic panels [4]. Nusselt numbers ranged from 3.2 to 68.3 ($1 \times 10^7 < Ra < 1 \times 10^{12}$) for one case, 13.3 to 56.6 ($1 \times 10^7 < Ra < 8 \times 10^9$), and 13.7 to 57.4 ($1 \times 10^7 < Ra < 8 \times 10^9$) for the final case. Calay et al. looked at heat transfer within rectangular enclosures with different boundary conditions [5]. Nusselt numbers ranged from 7.8 at a $Ra$ of $3.9 \times 10^8$ for one configuration to a high of 74.5 at a $Ra$ of $6.5 \times 10^9$ for another configuration. Terekhov and Terekhov modeled heat transfer in a vertical container with various numbers and lengths of fins attached to the hot side [6]. Rao analyses a vertical channel air heater to compare convective and radiative heat transfer [7].

Since this experiment, at its foundation, bridges two fields, it is important to look at what previous scholars have done in archaeology as well. Kretschmer preformed one of the most in-depth investigations of hypocaust systems in the early 1950s. In his article [8], he chronicles temperatures in a replica system that he built. Hüser [9] continued to perform experiments on Kretschmer’s replica focusing on the raised floor and analyzing the modern applicability of the hypocaust. Jorio [10] provides detailed information about the hypocausts of the Pompeian baths (Stabian, Forum, and Central) and then analyzes the heat losses from four rooms in the Stabian Baths. Rook [11] utilizes thermodynamic equations to come up with an approximation of how much fuel the Welwyn Roman bath consumed. Baatz [12] investigated the effectiveness of canal heating and compared this to the hypocaust system. Basaran preformed multiple numerical analyses on the hypocaust. The first [13] used numerical heat transfer to investigate the heating system of the Small Baths at Phaselis in Turkey. The second [14] used CFD to analyze the heating system of a bath house in Metropolis. The third and final [15] is, in essence, a summary of the first two. Yegül and Couch used temperature probes in the replica bath to analyze the heat losses [2]. The issue with all of these studies, however, is no one determines the convective heat transfer coefficient of the tubuli which is where this study fits in.

2. EXPERIMENTAL SETUP

The basic setup for this experiment (Fig. 3a) was a section of wall placed above a plenum which houses the heater. A wooden box faced with cement board on the inside and covered with both batt and rigid insulation encased these. A volume of air above the plenum and next to the wall section was meant as a faux room to prevent the direct conduction between the wall and the outside. It was important to maintain material consistency as much as possible between this case study and the wall we were replicating. The tubuli used in the bath were custom made and thus unattainable. Superior Clay terracotta flue liners were similar in size, shape, and material to the tubuli in the replica bath. The tubuli were prismatic and were 200 mm wide by 120 mm deep by 250 mm high. The flues were 216 mm wide by 114 mm deep by 607 mm high but the shorter edges were semi-circular instead of straight. Both the tubuli and the flues are of modern terracotta, however, the specific composition is most likely not the same. Six of the flues in an array three wide by two high with the rounded sides facing each other made up the main wall component. Lime mortar and marble tiles then covered the side facing the room substitute.

The initial heater was a Watlow 300 W finned strip heater. A thermocouple located above and behind the heater, measuring what will be referred to as the plenum temperature, was attached to a controller. The peak temperature with the 300 W heater was 40°C which was insufficient. A larger 750 W finned strip heater was more appropriate as its peak temperature was 160°C.
One final set of adjustments had to be made to keep the mass flow rate steady. A 31.8 mm hole was drilled in the front door of the plenum and the outside two flues were capped with rigid foam. This ensured the correct draw weight for the heater.

There were 16 K-type (Chromel-Alumel) thermocouples used for measurement. All were located in the centre two flue liners to minimize the effect of the edges. They were broken down into two groups of eight — one cluster located half way up the lower flue liner (referred to as the bottom cluster) and the other half way up the upper flue liner (referred to as the top cluster). In each group, the spread was about 50.8 mm and thermocouples were located at specified depths. The eight depths were: the middle of the flue (capturing the air temperature), on the inner surface of the flue, one-third of the flue wall thickness, two-thirds of the flue wall thickness, on the outer flue surface, half way through the mortar, on the inner surface of the marble, and finally, half way through the marble (refer to Fig. 3b).

The thermocouple placement was difficult because, unlike in many experiments, they had to be inserted during construction. This posed two major problems. The first was ensuring that the thermocouples did not move without drastically affecting the thermal properties. A combination of silicone and tape helped, however, some of the thermocouples may have moved slightly even with this procedure. The second was ensuring that no two thermocouples were directly on top of one another while still ensuring a tight clustering. Since all the materials were opaque and each layer erases the landmarks, an approximate circular pattern was employed.
An unexpected problem arose with the thermocouples on the inner surface of the flue next to the rising air. Even though the thermocouples were encased in concrete when they were mounted, the readings indicated that they were measuring the air temperature because they mirrored the fluctuations in the free stream readings. To test this hypothesis, a FLIR infrared camera took a sample of temperature readings. The infrared measurements proved to be drastically different from the bare wire thermocouple readings and closer to the expected data, so a K-type infrared thermocouple mounted on a rod in the middle of the flue replaced the surface thermocouple.

The data acquisition setup was uncomplicated. After the thermocouples measured the signals, they passed through an AD595 amplifier and captured by a NI PCI-6024E DAQ. The long time frame of this experiment, however, made continuous data sampling challenging. A custom LabVIEW virtual instrument (VI) program sampled 30 seconds worth of data at 200 Hz every 5 minutes. The VI then averaged the data and plotted these points on a continuous graph.

2.1. Calibration

The thermocouple calibration had to be completed in situ. The lower point was ascertained by allowing the system to reach room temperature. To obtain a higher point, insulation was placed over the marble and the inlet and outlet were both sealed with insulation. A fan was installed on the top which draws air from one flue and blows it back down another. The result was that the temperature equalized throughout the thickness of the wall. This provided the second point which allowed for the calculation of the calibration slopes and offsets.

3. RESULTS

Once the methodology and equipment were satisfactory, the experiment was run on five plenum temperatures (60°C, 70°C, 80°C, 90°C and 100°C) for each thermocouple cluster. Three temperatures were also duplicated — two for the bottom cluster and one for the top — to determine repeatability. The reason we chose the plenum temperature is that it is the only controllable variable. Most experiments measuring CHTCs use the wall temperature or heat flux as their reference but neither are applicable here. The wall temperature is constantly changing as it is warming up and there is no external heat flux outside of the heat from the rising air.

Since data collection took over twelve hours, the system did not have the opportunity to cool completely between runs. To compensate, on cool days, the data collection only began after the system warmed up. Even with this precaution, though, the starting temperatures of the individual thermocouples were never the same. However, the resulting CHTC values differed by a maximum of 4.7 % between comparative runs.

Each run produced eight data curves — one for each thermocouple in one of the two clusters. Simultaneous measurement of the two data clusters was not possible because of technical limitations of the setup.

For the temperature measurements there are three sources of error: the thermocouples themselves, the amplifier, and the DAQ. The standard K-type thermocouple has an accuracy of ±1.1°C, and the infrared has an accuracy of 2 % or a maximum of ±1.2°C. According to the literature, the amplifier used has an error of ±1°C, while the DAQ has a maximum error of ±3.8764 mV which converts to ±0.38°C. Therefore the maximum total error, using the root-sum-square methodology, is ±1.53°C for the K-type thermocouple and ±1.61°C for the infrared thermocouple.

Figure 4 shows the temperature data from the bottom cluster when the plenum was 90°C and Fig. 5 shows the temperature data from the top cluster at the same plenum set point. The first thing that immediately becomes apparent is the difference between both the individual temperatures and their trend. With the bottom cluster, the air temperature reached a fairly steady value after approximately three hours whereas,
the top cluster never stabilizes but rather keeps increasing throughout the entire 7 hour time period. This was most likely because the top cluster was only 305 mm from the exit so there was a competing downdraft from the room which would cause turbulence. Another interesting difference between the top and bottom clusters is the surface temperature. In the bottom cluster the surface temperature is less than 1°C hotter than the next thermocouple reading whereas with the top cluster the surface temperature is more than 4°C away and only just over 2°C from the air temperature. This happened with all the top cluster tests and, given the fact that the surface temperature readings started at roughly the same point, it cannot be a systematic sensor error. This disparity, as seen below, caused a significant increase in the CHTC as it raises flux values and, more importantly, decreases the temperature difference on which the CHTC is based.
The radiative heat transfer to the room might be a partial reason for the discrepancy between the surface temperatures of the top and bottom clusters. To test the plausibility of this, the system was run with the flues closed from external influences. While sealing the flues changes the fluid dynamics, the heat transfer mechanism is essentially the same. The results were definitive. With the flue sealed, the surface temperature was less than a degree from the next thermocouple in the wall.

4. CALCULATION OF COEFFICIENTS

The equation for convective heat transfer is:

\[ h = \frac{q''}{T_w - T_{\infty}}. \]  

(1)

The raw data provide \( T_{\infty} \) and \( T_w \) which leaves \( q'' \), the heat flux. There are two negative heat fluxes — conduction through the wall and radiation from the interior wall of the flue. Assuming that the thermal conductivities are uniform throughout their thickness, the transverse one-dimensional Fourier equation is:

\[ q''_{\text{con}} = k_{\text{eff}} \frac{T_1 - T_2}{\Delta x}. \]  

(2)

In this equation, \( k_{\text{eff}} \) is the effective thermal conductivity of the section of the wall, \( T \) is the temperature at a specified point, and \( \Delta x \) is the distance between the two points. To palliate the possible random error from using only two points, the best alternative was to calculate multiple fluxes. Effective thermal conductivities were calculated for eleven sets of data points — six using the surface as a reference and five using the marble as a reference — see Table 1 for classification.

<table>
<thead>
<tr>
<th>Surface-referenced heat fluxes</th>
<th>Marble-referenced heat fluxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface to ( \frac{1}{3} ) terracotta</td>
<td>( \frac{1}{3} ) terracotta to mid-marble</td>
</tr>
<tr>
<td>Surface to ( \frac{2}{3} ) terracotta</td>
<td>( \frac{2}{3} ) terracotta to mid-marble</td>
</tr>
<tr>
<td>Surface to terracotta/mortar interface</td>
<td>Terracotta/mortar interface to mid-marble</td>
</tr>
<tr>
<td>Surface to mid-mortar</td>
<td>Mid-mortar to mid-marble</td>
</tr>
<tr>
<td>Surface to mortar/marble interface</td>
<td>Mortar/marble interface to mid-marble</td>
</tr>
</tbody>
</table>

Table 1. Breakdown of heat fluxes.

The conduction heat fluxes were calculated for each time increment. Figure 6 shows the resulting conductive heat fluxes for the bottom cluster using the surface as the reference in the same sample plenum temperature above and Fig. 8, using marble as the reference. Figures 7 and 9 show the fluxes for the top cluster. First thing to note for both cases is how quickly the conduction steadies compared to the temperature. By 500 minutes, the flux in both clusters had stopped fluctuating appreciably. This means that, while the temperature was still climbing, the amount of heat going into the wall remained constant.

With the uncertainty in the temperature being high relative to the temperature differences in the wall, the uncertainty in the heat flux is large, despite the fact that two of the three variables are actually constants. For both clusters, therefore, the scattering of the heat fluxes can be attributed to experimental error. This is most readily apparent in Figs. 6 and 8 where the 1/3 TC curve is separated from the remaining curves. The distance between this thermocouple and the wall surface is just over 4 mm and is the smallest of any of the surface-referenced calculations. This means that the heat flux calculation magnifies any error in the 1/3
TC thermocouple reading and gives the appearance of an alternate heat flux value. In addition, there is no discernable pattern to the spread, however, the closer together the thermocouples are, the more likely it is for the resulting fluxes to be more askew because the random error grows relative to the temperature difference.

Fig. 6. Heat fluxes from bottom cluster with the surface temperature as the reference point.

Fig. 7. Heat fluxes from top cluster with the surface temperature as the reference point.

The radiative heat transfer was not easy to estimate. In the Roman bath heating system, each flue would be covered by a horizontal pipe which would take the exhaust gas to a chimney and vent it outside whereas in the experimental setup, the flue was open to the room. This presents two possibilities for the radiation from the inner wall. In the bath, the radiation contained within the flue would not transfer much heat as the walls are within 2°C of each other and there would be little radiation. In the experiment, however, the radiation could diffuse throughout the room thereby cooling the surface.
Fig. 8. Heat fluxes from bottom cluster with the mid-marble temperature as the reference point.

Fig. 9. Heat fluxes from top cluster with the mid-marble temperature as the reference point.
For the purposes of this setup, the heat transfer due to radiation was initially ignored. Therefore, the CHTC was obtained by dividing the heat flux by the difference between the surface temperature and the bulk temperature, refer to Eq. (1). An average was taken as well, excluding those CHTCs that were obviously skewed, namely the one calculated from the heat flux between the mortar/marble interface and the mid-marble thermocouples. The results were then graphed versus time and are shown in Figs. 10–13. Figure 10 shows the results from the bottom cluster for a plenum temperature of 90°C using the surface-referenced heat fluxes and Fig. 12 the marble-referenced heat fluxes. Figures 11 and 13 display the CHTC breakdown for the top cluster. The patterns were nearly identical as those for the fluxes above, particularly when the results steadied. The fluctuations in the CHTC came from the fact that the bulk temperature was never constant.

Fig. 10. CHTCs using heat fluxes from bottom cluster with the surface temperature as the reference point.

Fig. 11. CHTCs using heat fluxes from top cluster with the surface temperature as the reference point.
There is a considerable difference in the CHTCs from the bottom and top clusters — nearly a factor of ten. The average of the averages for various plenum temperatures are shown in Table 2.

The uncertainty determined for the CHTC (and $Nu$ latter) appears to be significant. While theoretically correct, however, we feel this does not accurately portray the experimental results. The uncertainty arises largely from the fact that the small temperature differentials in the heat flux calculations are of the order of the thermocouple errors, as mentioned earlier. When we compared the results of multiple runs for the same conditions, the standard deviation of the temperature data was less than 0.24°C. Given these runs, done on different days, have this small a standard deviation suggests good experimental repeatability and that the calculated errors, while possible, were not realized.
The CHTCs for the bottom cluster are close to other values for natural CHTCs for air, however, the values from the top are considerably higher. When compared with the sealed flue mentioned above for which the average CHTC was 7.7 W/m²·°C for a trial when the plenum temperature was 90°C, the data from the top cluster become suspect. Also, for the case under investigation, it is a safe assumption that most of the terracotta tubes in the walls of the bath would be exposed to scenarios much like the lower cluster or the sealed top cluster and not to the open top cluster. For this reason, the data from the bottom cluster will only be considered as accurate within the narrow confines of the initial question. Furthermore, generally, it should be noted that the CHTCs do not vary as a function of plenum temperature. This is somewhat surprising as the expectation is for the CHTC to increase somewhat as the air temperature rises. In fact, if anything, the CHTC peaks when the plenum temperature is 80°C.

5. DISCUSSION

In order to compare these results to the studies mentioned in the introduction, the Nusselt number is computed as:

\[ \overline{Nu}_l = \frac{hL}{k}. \]  

(3)

In this equation, \( h \) is the CHTC, \( L \) is the characteristic length, and \( k \) is the thermal conductivity of the air at, in this case, the bulk temperature. For this particular setup, the best choice for the characteristic length is the height of one flue or 607 mm. This is for a number of reasons. One, this arrangement is most similar to channel flow which uses height as the length scale. Two, since there are two clusters of thermocouples, each centred on a flue, it is the natural divisor in the system. Three, that distance allows the Nusselt number to be typical of the flue. The other key comparator is the Rayleigh number:

\[ Ra = \frac{g \beta (T_w - T_\infty)L^3}{\nu \alpha}. \]  

(4)

In this equation, \( g \) is the gravitational acceleration, \( \beta \) is the thermal expansion coefficient for air, \( T_w \) is the surface temperature, \( T_\infty \) is the bulk temperature of the air, \( L \) is the characteristic length, \( \nu \) is the kinematic viscosity of the air, and \( \alpha \) is the thermal diffusivity of the air. All air properties were taken at the bulk temperature. Since, however, for this case the wall temperature is lower than the bulk temperature so the Rayleigh number is negative. Another notable difference is the temperature at which the thermal properties are calculated. Unlike most studies, the driving force of this study is the air so the film temperature is not applicable. Therefore, the bulk temperature was used to calculate the properties. The Rayleigh and Nusselt numbers for the bottom cluster are given in Table 3.

<table>
<thead>
<tr>
<th>Plenum Temperature (°C)</th>
<th>Bottom cluster CHTC (W/m²·°C)</th>
<th>Top cluster CHTC (W/m²·°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>7.2 ± 10.0</td>
<td>63.6 ± 138</td>
</tr>
<tr>
<td>70</td>
<td>7.0 ± 8.9</td>
<td>64.6 ± 104</td>
</tr>
<tr>
<td>80</td>
<td>7.6 ± 8.6</td>
<td>69.8 ± 140</td>
</tr>
<tr>
<td>90</td>
<td>6.8 ± 9.2</td>
<td>63.2 ± 131</td>
</tr>
<tr>
<td>100</td>
<td>6.2 ± 8.4</td>
<td>60.2 ± 86.2</td>
</tr>
</tbody>
</table>

Table 2. Average CHTC for both clusters.
The first thing that becomes clear is that there is no relationship between the Rayleigh and Nusselt numbers. This is most likely because the Rayleigh, or more specifically the Grashof, number is a comparative measure between buoyancy and viscous forces and, since the air is actually cooled by the walls, it does not capture the mechanics of this setup even though it is a natural convection system.

If one just looks at the Nusselt numbers, however, there are a few things to note. The pattern of peaking at 80°C present in the CHTCs is more exaggerated here. These Nusselt numbers are more than double than all but one of the cases mentioned above. Interestingly, the one case (the channel flow studied by Hatami and Bahadorinejad) that is higher is the closest comparison. When compared to these, the Nusselt numbers here are about half. This setup, which is approximately two millennia old in design, is, therefore, half way between the comparable modern equivalents which may be surprising. In essence, the system of terracotta pipes is not only incredibly effective for its age, it matches modern counterparts.

6. CONCLUSION

The goal of this project was to determine the convective heat transfer coefficient (CHTC) for terracotta flues designed to replicate the heating system inside an ancient Roman bath. The unique aspect about these experiments was that the air was heated before being exposed to the faux wall and then allowed to rise through the flues, thereby warming the flue walls. The five plenum temperatures were tested (60°C, 70°C, 80°C, 90°C, 100°C) and we found that the CHTC was relatively insensitive to plenum temperature. The average CHTC was 7.0 W/m²°C. Furthermore, there was no relationship established connecting the Nusselt number which ranged from $1.3 \times 10^2$ to $1.6 \times 10^2$ with the Rayleigh number which ranged from $-1.7 \times 10^8$ to $-1.5 \times 10^8$.

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REFERENCES