RESEARCH ON THEORY AND APPLICATIONS OF VIRTUAL TOLERANCE

Wang Xiaohui¹, Ren Shouhua¹,²

¹ School of Mechanical and Engineering, Taiyuan University of Science and Technology, Taiyuan, Shanxi, China.
² Zhengzhou Science Research & Design Institute State Administration of Grain Reserve, Zhengzhou, Henan, China.
E-mail: houmawxh@sina.com; shouhua000@163.com

Received January 2012, Accepted May 2012
No. 12-CSME-10, E.I.C. Accession 3331

ABSTRACT

Tolerance where the upper deviation is smaller than the lower deviation is defined as virtual tolerance. The essence of virtual tolerance is illustrated in this paper: the absolute value of virtual tolerance is the amount of error compensation whose range is the difference between the upper deviation and the lower deviation. This value can be described as virtual tolerance and provides a basis for the theory of tolerance. Based on this theory, we can unify calculation principles and methods regarding various types of assembly dimension chains, accurately calculate the tolerance of dimensional chains for assembly with compensation assemblies, and put forward a general calculation method for false waste product intervals.

Keywords: virtual tolerance; error compensation; tolerance calculation; calculation of false waste product interval.

TOLÉRANCE VIRTUELLE : THÉORIE ET APPLICATIONS

RÉSUMÉ

On définit la tolérance virtuelle comme la tolérance à la déviation la plus large par rapport à la déviation la plus restreinte. L’essentiel de la tolérance virtuelle est illustré dans cet article. La valeur absolue de la tolérance virtuelle est la valeur de l’erreur de compensation, l’erreur étant la différence entre la plus grande déviation et la plus petite. Cette erreur peut être décrite comme la tolérance virtuelle, et elle est à la base de la théorie de la tolérance. En se basant sur cette théorie, on peut donner un caractère commun aux principes de calcul et aux méthodes à l’égard des chaînes d’assemblage de différents types, et calculer précisément la tolérance avec compensation des chaînes d’assemblage, et mettre de l’avant une méthode de calcul général pour les intervalles de fausse perte de matériel.

Mots-clés: tolérance virtuelle ; erreur de compensation ; calcul de la tolérance ; calcul d’intervalle de fausse perte de matériel.
1. INTRODUCTION

1.1. Virtual tolerance resulting from the calculation of dimension chain

Figure 1 shows a gear assembly that demonstrates a dimension chain calculation. The image is taken from the Appendix D of China National Standards GB5847–86 [1]. In order to ensure that the gear turns easily and no obvious axial movement occurs, the requirement for the end play is \( A_0 = 0^{+0.35}_{-0.1} \). The relationship of the dimensions in Fig. 1 is:

\[
A_0 = A_3 - A_1 - A_2 - A_4 - A_5. \tag{1}
\]

Now we need to confirm the dimensions and the tolerances of the relevant assembly. Firstly, calculate the axial dimensions and the tolerances of the other parts according to economical accuracy:

\[
A_1 = 30^{0}_{-0.2}; A_2 = 5^{0}_{-0.1}; A_3 = 43^{+0.2}_{0}; A_4 = 3^{0}_{-0.05}.
\]

Then plug the above dimensions and tolerances into Eq. (1):

\[
0^{+0.35}_{-0.1} = 43^{+0.2}_{0} - 30^{0}_{-0.2} - 5^{0}_{-0.1} - 3^{0}_{-0.05} - A_5. \tag{2}
\]

From Eq. (2), we see: 0.35 = 43.2 – 29.8 – 4.9 – 2.95 – 0.1 = 43 – 30 – 5 – 3 – 0.2. The result is \( A_{5_{\text{min}}} = 5.2; A_{5_{\text{max}}} = 4.9 \), namely, \( A_5 = 5^{0.1}_{+0.2} \). Based on this result, we may find that the upper deviation of \( A_5 \) is smaller than its lower deviation \( (A_{5_{\text{min}}} > A_{5_{\text{max}}}) \). The tolerance value of \( A_5 \) is negative:

\[
\delta_{A_5} = -0.1 - 0.2 = -0.3.
\]

When the upper deviation is smaller than the lower deviation, it is defined as virtual tolerance.

1.2. Interpretation of virtual tolerance

Any dimension of rigid assembly cannot meet the demand of virtual tolerance put forward above. That is why the current tolerance theory considers that virtual tolerance with the upper deviation smaller than the lower deviation does not exist or is meaningless. Thus, the virtual tolerance defined above goes against the current theory of tolerance.

However, this paper argues that virtual tolerance must be accounted for. Firstly, because virtual tolerance can now be calculated, it must own its particular meaning. Secondly, many dimensions, can in fact meet the definition of virtual tolerance under certain conditions. For instance, a spring that alters its length under
different pressure can meet the demand of virtual tolerance within a certain range. In addition, only when the sealing surface of a sealing gasket can meet the demand of plane virtual tolerance within a certain range under a certain pressure, can the function of sealing be attained. This occurs when it is made free of the clearance between the sealing surface and the metal faying surface.

To sum up, we should recognize virtual tolerance broadly and explore its nature to develop and perfect the theory of tolerance so that more applications can be found in practice.

2. ESSENCE OF VIRTUAL TOLERANCE

2.1. Virtual tolerance referring to error compensation

In Fig. 2, spring 2 and base plate 3 are put into the groove of the bearer 1; the height of the groove $A_1 = 100 \pm 0.1$ and that of the base plate $A_3 = 40 \pm 1$.

![Fig. 2. Dimension of the spring length and virtual tolerance: 1-bearer; 2-spring; 3-base plate.](image)

Next we must analyze what qualification the dimension and the tolerance of $A_2$ should have if the following relation is to be ensured:

$$0 = A_1 - A_2 - A_3. \tag{3}$$

Plug $A_1 = 100 \pm 0.1$ and $A_3 = 40 \pm 1$ into Eq. (3):

$$0 = (100 \pm 0.1) - A_2 - (40 \pm 1). \tag{4}$$

From Eq. (4):

$$0 = 100.1 - A_{2\min} - 39, \text{ then } A_{2\min} = 61.1, \quad 0 = 99.9 - A_{2\max} - 41, \text{ then } A_{2\max} = 58.9.$$ 

Namely, $A_2 = 60.0_{+1.1}^{-1.1}$.

The tolerance of $A_2$ is virtual tolerance. According to the definition of tolerance, the height of the spring should be no more than 58.9 and no less than 61.1. Obviously, it is easy to meet this demand as the height of the spring can be compressed continuously from 61.1 to 58.9, and it can meet the demands of virtual tolerance. Thus, for a dimension that has the demand of virtual tolerance, it actually demands that the dimension can vary in the range of virtual tolerance continuously. On the other hand, the demand of virtual tolerance within the range can be met only if the dimension can vary in the range continuously.

Therefore, error is not allowed for dimensions that have the demand of virtual tolerance. In the meantime, all dimensions within the range of virtual tolerance should be met by it. In other words, it demands that the dimension be able to compensate the dimension error within the range of the virtual tolerance. In the
instance above, while \( A_1 \) and \( A_3 \) are any dimensions within their ranges of tolerance, \( A_2 \) must compensate for the error, and the amount of compensation is \(|−1.1 − 1.1| = 2.2\) with the range of it from 58.9 to 61.1.

To sum up, the essence of virtual tolerance is that dimensions are needed to compensate for errors. The absolute value of this is the amount of the error compensation. The dimension range between the upper and the lower deviation is the range of the error compensation.

2.2. Relationship between virtual tolerance and tolerance

After the concept of virtual tolerance is established, tolerance will obtain broader significance: its value can be any real number; when it is positive, it allows a dimension to have error; when it is zero, no error is allowed; when it is negative, it demands that the dimension compensate the error. What then is the relationship between virtual tolerance and conventional tolerance?

2.2.1. Virtual tolerance being unified with tolerance

Virtual tolerance is a part of tolerance and has the property of tolerance, as the extension of the definition of tolerance does not change its basic properties. In Fig. 1, while \( A_1, A_2, A_3 \) and \( A_4 \) were distributed appropriate tolerances, \( A_5 \) was also distributed tolerance (virtual tolerance).

Virtual tolerance is unified with tolerance in several other points. Firstly, virtual tolerance can ensure that the dimension can be no more than the upper deviation and no less than the lower deviation. As we have seen, dimension \( A_2 = 60^{−1.1}_{+1.1} \) in Fig. 2 can ensure that the dimension is no more than 58.9 (when \( A_1 = 99.9, A_3 = 41 \) and the spring is maximally compressed), and also no less than 61.1 (when \( A_1 = 100.1, A_3 = 39 \) and the spring is minimally compressed).

Secondly, the value of virtual tolerance is still the difference between an upper deviation and a lower deviation, only with a negative result. As with tolerance, the smaller the virtual tolerance is, the harder the dimension can be ensured. For the dimensions \( 10^+0.2, 10^+0.1, 10^+_0.1 \) and \( 10^-0.2, 10^-0.5 \), the values of tolerance become smaller and smaller in proper sequence, thus more and more difficult to ensure. The comparison between the first two dimensions is obvious. The (virtual) tolerance of the third dimension is \(-0.1\) and the demanded amount of error compensation is 0.1. On the other hand, the (virtual) tolerance of the fourth dimension is \(-0.3\) and the demanded amount of compensation is 0.3; thus, it is more difficult to ensure the last dimension than the third dimension.

2.2.2. Virtual tolerance being an opposite concept to tolerance

Tolerance restricts error while virtual tolerance compensates error. Any dimension within the range between upper deviation and lower deviation can fit the design requirements. As to a dimension with virtual tolerance, however, it demands either that the dimension compensate error in the range of the virtual tolerance, or that it fit any dimension within the range of the virtual tolerance. This is the fundamental difference between virtual tolerance and tolerance. In addition, to a dimension with tolerance, the larger the range between upper deviation and lower deviation is and the larger the allowable error is, the easier it will be to ensure the dimension. Regarding a dimension with virtual tolerance, however, the case is just the opposite.

3. APPLICATIONS OF VIRTUAL TOLERANCE

When virtual tolerance is used to describe the amount and the range of error compensation, it will be more convenient for us to analyze and solve some problems relevant to error compensation, which were previously difficult to deal with.
3.1. Applications on calculation of assembly dimension chain

3.1.1. Calculating methods of assembly dimension chain being unified on the basis of the concept of virtual tolerance

As for the assembly dimension chain with a compensation assembly, conventional methods cannot be used due to virtual tolerance. Therefore, a number of particular formulas have to be deduced according to different assembly situations to get a result, which adds numerous steps and makes the calculation complicated.

After the explanation of virtual tolerance has been made, it will not be necessary to avoid virtual tolerance and conventional formulas can be used in the calculation of assembly dimension chains. If the result of a certain tolerance is virtual tolerance, we thereby can determine the amount and the range of the error compensation. Therefore, the principles and methods of calculating assembly dimension chains are unified, which is suitable for computer aided calculation.

In the instance of Fig. 1, \(A_5 = 5_{-0.1}^{+0.2}\), meaning that the amount of error compensation is 0.3, and the range of error compensation is from 4.9 to 5.2. Using any assembly method should ensure this virtual tolerance indirectly.

1. Fitting: The fundamental point is that assembly precision is attained by fitting \(A_5\) in the process of assembly (tolerance of the closing link). In order to ensure fitting, the initial dimension of \(A_5\) must fit in the greatest entity dimension within the range of virtual tolerance, and the tolerance should extend toward the direction of the greatest entity.

The greatest entity dimension of \(A_5\) between 4.9 and 5.2 is 5.2. If its tolerance is 0.1, the initial dimension will be \(A_5' = 5.2_{0}^{+1}\).

The maximum fitting allowance \(Z_{\text{max}} = 5.2 + 0.1 - 4.9 = 0.4\)

The minimum fitting allowance \(Z_{\text{min}} = 5.2 - 5.2 = 0\).

2. Adjustment: The fundamental point here is to classify the dimensions within the range of virtual tolerance (4.9–5.2) of \(A_5\) into several groups according to their sizes. When assemble, we choose a certain dimension of \(A_5\) to meet the assembly precision according to the actual size of the other component loops.

In the assembly of a certain gear component shown in Fig. 1, the specific dimensions from \(A_1\) to \(A_4\) have been decided, then \(A_0 = (A_3 - A_1 - A_2 - A_4) - A_5 = A_5 - A_5\), if \(\delta_{A_5} = 0.1\), then \(\delta_{A_5} = \delta_{A_0} - \delta_{A_5} = 0.25 - 0.1 = 0.15\).

In other words, if the error of \(A_5\) is within 0.15, it can ensure the assembly precision with an \(A_5\), then the size of space is 0.15. The specific dimensions are as follows: \(A_5 = 4.9_{-0.1}^{0}, 5.05_{-0.1}^{0}, 5.2_{-0.1}^{0}, 5.35_{-0.1}^{0}\).

The result by the extremum method based on the concept of virtual tolerance equals the result derived from the conventional method, yet through a much more simplified process.

3.1.2. Calculating dimension chains with compensation assembly with probabilistic method

For dimension chains that contain many component loops, it is economical and reasonable to use a probabilistic method to calculate tolerance. Without the theory of virtual tolerance, the tolerance calculated with the tolerance formula of probabilistic method must be an imaginary number as for an assembly dimensional chain with compensation assembly. Therefore, we can directly use the calculating formula of dimension chains, with the concept of virtual tolerance, to calculate the tolerance of dimension chains of this type that we could not do with probabilistic method before.
Now we can calculate the dimension chain in Fig. 1 with the probability method, convert all the dimensions into mean size and plug into Eq. (3):

$$A_5 = 43.1 - (29.9 + 4.95 + 2.975) - 0.225 = 5.05.$$  

Assuming the production condition is for mass production, a stable technological process, and that the dimensions of components approach normal distribution, and we can choose relative asymmetric coefficient $e = 0$, relative distribution coefficient $k = 1$. Then:

$$\delta_{A_5} = \sqrt{0.25^2 - 0.2^2 - 0.1^2 - 0.2^2 - 0.05^2} = 0.173i.$$  

The tolerance calculated is an imaginary number, the square of which is negative. It means that the range of the compensation is equal to the value of the imaginary number, with the amount of the compensation being 0.173.

$$A_5 = 5.05 \pm 0.173/2 \approx 5.05 \pm 0.09,$$

the amount of the compensation is 0.173, and the range is from 4.96 to 5.14.

As no explanation was given for virtual tolerance before, in order to avoid virtual tolerance, we had to follow many steps to get a result. First, the synthesis tolerance of $A_1$, $A_2$, $A_3$ and $A_4$ obtained with conventional probabilistic method is:

$$\delta_C = \sqrt{0.2^2 + 0.1^2 + 0.2^2 + 0.05^2} = 0.30.$$  

Then we calculate the amount of the compensation with the extremum method, $0.3 - 0.25 = 0.05$.

Seen from the above result, the conventional method is a hybrid algorithm of the probability method and the extremum method. The amount of the compensation obtained with this method is too small to fit the assembly precision. This is why the China National Standards do not recommend the probability method to calculate the assembly dimension chain with a compensation assembly. Nonetheless, the result obtained with the basic formula of a dimension chain in this paper is 0.173, which is between the results obtained with the probability method and the extremum method. Thus it is both economical and reliable.

Figure 3 shows a comparison of the amounts of the compensation (vertical coordinate) of $A_5$ with three methods when the tolerances of $A_1$, $A_2$, $A_3$ and $A_4$ have been determined and the tolerance of the closing link (horizontal coordinate) varies.

As seen in Fig. 3, when the assembly precision varies, the range of compensation of the compensation assembly obtained with the method discussed in this paper is always within the extremum method and the probabilistic method, but when put into actual production, it is more economical than results obtained with the extremum method and more reliable than that with the probabilistic method.

### 3.2. General calculating formula of false waste products intervals

Applying the concept of virtual tolerance, it will be very convenient to determine the false waste product intervals in the machining process and to obtain general calculating formulas.

In the machining process, if the designing dimension $A$ of a component is guaranteed by three process dimensions $B$, $C$ and $D$ indirectly, and:

$$A = B + C - D.$$  

(5)

Add tolerances to Eq. (5):

$$A_{ei(A)} = B_{ei(B)} + C_{ei(C)} - D_{ei(D)},$$  

where $es$ is the upper deviation of the relevant dimension and $ei$ is the lower deviation. If dimensions $B$, $C$ and $D$ can meet the demand of Eq. (6), then the designing dimension can be definitely ensured.
Fig. 3. Compensation dosage in different methods.

Exchanging the upper and the lower deviation of \( B \) as well as \( D \) in Eq. (6), the virtual tolerance will occur:

\[
A_{ei(A)}^{es(A)} = B_{es(B)}^{ei(B)} + C_{ei(C)}^{es(C)} - D_{es(D)}^{ei(D)}. \tag{7}
\]

Equation (7) is different from Eq. (6) in “probably”. Process dimension \( B \) and \( D \) can probably be any size in the virtual tolerance. The calculated value of the tolerance of process dimension \( C \) by Eq. (7) can probably meet the designing dimension \( A \) and the demand of its tolerance.

In the instance above, if the designing dimension \( A_{ei(A)}^{es(A)} = 40^{+0.3} \), the process dimension \( B_{es(B)}^{ei(B)} = 100^{+0.1} \), \( D_{es(D)}^{ei(D)} = 80^{0}_{-0.1} \), plug them into Eq. (6) and we get process dimension \( C_{ei(C)}^{es(C)} = 20^{+0.1} \). It means that the dimension and the tolerance of \( A \) can be definitely met if we process the work piece according to the above dimensions \( B, C \) and \( D \) as well as their tolerances.

If we calculate according to Eq. (7), namely by changing the tolerances of \( B \) and \( D \) into virtual tolerance, then:

\[
40^{+0.3} = 100^{0}_{+0.1} + C_{ei(C)}^{es(C)} - 80^{0}_{-0.1}. \tag{8}
\]

From Eq. (8), \( C_{ei(C)}^{es(C)} = 20^{+0.3}_{-0.1} \), this means that \( C = 20^{+0.3}_{-0.1} \) that fits Eq. (8) can probably fit designing dimension \( A \). Namely, the false waste products interval of process dimension \( C \) is \( 20^{+0.3}_{+0.1} \) and \( 20^{0}_{-0.1} \).

Similarly, change the tolerance of \( B \) and \( C \) in Eq. (6) into virtual tolerance and the false waste product intervals of process dimension \( D \) can be calculated.

4. CONCLUSIONS

1. Tolerance can be any real number. Tolerance is continuous. Virtual tolerance belongs to the category of tolerance and it describes the amount and the range of error compensation.

2. Calculating theories and methods of all sorts of assembly dimension chains will be unified and the process of calculating will be greatly simplified on the basis of the concept of virtual tolerance. Teaching will become more efficient and it will be suitable for computer aided design.

3. The general formula of false waste product intervals can be set up based on the concept of virtual tolerance.
4. To calculate accurately the amount of error compensation and the false waste product intervals with probabilistic method, it must be based on the concept of virtual tolerance. It will provide more accurate data for the course of production.

With the development of research on virtual tolerance, its application will be extended widely and exert great influence on the theory of modern precision designing.

ACKNOWLEDGEMENTS

This project is supported by National Natural Science Foundation of China (Grant No.51175360), and it is also supported by ShanXI Provincial Natural Science Fund (Grant No.2009011029–1).

REFERENCES