ABSTRACT

Net energy stored ($Q_{net}$) and the discharge time of Phase Change Material ($t_{PCM}$) in a solar system, are important conflicting objectives to be optimized simultaneously. In the present paper, multi-objective genetic algorithms (GAs) are used for Pareto approach optimization of a solar system using modified NSGA II algorithms. The competing objectives are $Q_{net}$ and $t_{PCM}$ and design variables are some geometrical parameters of solar system. It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of solar system can be discovered. These important results can be used for better design of a solar system.

Keywords: multi-objective optimization; solar System; PCM; genetic algorithms.

OPTIMISATION MULTIOBJECTIF DE PARETO D’UN SYSTÈME DE STOCKAGE D’ÉNERGIE SOLAIRE THERMIQUE UTILISANT DES ALGORITHMES GÉNÉTIQUES

RÉSUMÉ

L’énergie nette ($Q_{net}$) stockée et le temps de changement de phase du matériau ($t_{PCM}$) dans un système solaire sont d’importants objectifs contradictoires à optimiser simultanément. Dans cet article, on se sert des algorithmes génétiques (AGs) à objectifs multiples pour l’optimisation PARETO d’un système solaire utilisant des algorithmes NSGA II modifiés. Les objectifs concurrents sont $Q_{net}$ et $t_{PCM}$, et les variables de design sont des paramètres géométriques du système solaire. Il est démontré que d’importantes et intéressantes relations, tels que les principes utilisés en design optimal associés à la performance du système solaire, peuvent être découvertes. Ces résultats intéressants peuvent être appliqués dans une conception améliorée d’un système solaire.

Mots-clés : optimisation multiobjectif; système solaire; PCM; algorithmes génétiques.
1. INTRODUCTION

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as to find a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. There are many calculus-based methods including gradient approaches to single objective optimization and are well documented in [1]. However, some basic difficulties in the gradient methods, such as their strong dependence on the initial guess, cause them to find local optima rather than global ones. Consequently, some other heuristic optimization methods, more importantly Genetic Algorithms (GAs) have been used extensively during the last decade. Such nature-inspired evolutionary algorithms [2] differ from other traditional calculus based techniques. The main difference is that GAs work with a population of candidate solutions not a single point in search space. This helps significantly to avoid being trapped in local optima [3] as long as the diversity of the population is well preserved. Such an advantage of evolutionary algorithms is very fruitful to solve many real-world optimal design or decision making problems which are indeed multi-objective. In these problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front [4] for multi-objective optimization problems.

Using thermal storage systems to gain hot water is becoming very popular as a result of limited fossil fuel sources and the requirement of protecting environment related to not polluting characteristics of this kind of energy. Many studies have been done to calculate transferred heat within Phase Change Material (PCM) and net heat gain by means of whole system. Ahmet Koca et al. [5] developed an experimental system with analysis of energy and exergy for a latent heat
storage system with PCM. Canbazoglu et al. [6] developed an experimentally investigation for calculating the midpoint tank temperature with sodium thiosulfate pentahydrate PCM. Varol et al. [7] made a series of predictions by using three different soft computing techniques to introduce an efficient method for calculating useful energy. Aghbalou [8] dealt with the exergetic optimization of a solar thermal energy system which consisted of a solar collector and a rectangular water storage tank that contains a PCM distributed in an assembly of slabs. Mettawee and Assassa [9] used paraffin wax as PCM in an experimental study to investigate the performance of a compact PCM solar collector based on latent heat storage.

In this paper, multi-objective genetic algorithms are used for Pareto approach optimization of solar systems. The objective functions are net energy stored \( Q_{net} \) and discharge time of PCM \( t_{PCM} \) and design variables are some geometrical parameters of solar system. It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of solar network can be discovered. Such important optimal principles would not have been obtained without the use of Pareto optimization approach.

2. MATHEMATICAL MODELING OF SOLAR SYSTEM

The schematic of thermal storage unit under analysis is shown in Fig. 1. It consists of a flat plate solar collector, a storage water tank with disodium hydrogen phosphate-dodecahydrate as the PCM content tank inside, a pipeline and several valves. Water temperature rises due to flowing through the collector when Sun is shining. Then, it goes into the water tank including the PCM tank. Heat energy transfers from warm water to the solid PCM, which its melting point is 29 centigrade degrees, and changes its phase into liquid. After that, water is ready to use. When the Sun descends, network water comes to the storage tank from bypass pipeline and receives thermal energy stored in fluid PCM during the day. In this process, water temperature increases while the PCM phase is changing. After that, water goes to pipeline to be consumed.

Energy analysis was carried out to evaluate the amount of heat energy stored by solar collector with the PCM. For a solar collector shown schematically in Fig. 1, the useful energy that increases water temperature during the flowing inside the collector before the water and PCM tank is represented in Eq. (1)
\[ Q_u = A_c F_R [S - U(T_i - T_a)] \]  

(1)

Where \( F_R \) can be calculated using Eq. (2) [10]

\[ F_R = \frac{\dot{m} C_p}{A_c U} \left[ 1 - e^{-\left(\frac{A_c U F}{\dot{m} C_p}\right)} \right] \]  

(2)

Collector efficiency factor, \( F' \) could be achieved from Eq. (3) [10]

\[ F' = \frac{U_i^{-1}}{W[U_L(D + (W - D)F)] + \frac{1}{\pi D_i h_f}} \]  

(3)

Total heat transfer coefficient is calculated from the expression below [10]

\[ U_i = \left[ \frac{N}{\left(\frac{344}{T_p}\right)^{N+1/3}} + 1 \right]^{-1} \]  

\[ + \frac{\sigma \left(T_a + T_p\right) \left(T_a^2 + T_p^2\right)}{[\varepsilon_p + 0.0425N(1-\varepsilon_p)]^{-1} + \frac{2N+f-1}{\varepsilon_g}] - N \]  

(4)

Net thermal energy gained with the absorber of collector may be obtained by multiplying instantaneous solar radiation by heat gain coefficient and effective absorptivity of solar collector which is shown in Eq. (5) below [10]

\[ S = HR(\tau \alpha) \]  

(5)

Effective absorptivity of solar collector can be defined with Eq. (6) [10]

\[ (\tau \alpha) = \rho \times \sum_{n=0}^{\infty} [(1-\alpha)\rho d]^n = \frac{\rho \alpha}{1-(1-\alpha)\rho d} \]  

(6)

\( R \) is the ratio of total radiation on tilted surface to that on plane of measurement and can be calculated as bellow [10]

\[ R = \frac{H_b}{H} \]  

\[ R_b + \frac{H_d}{H} \]  

(7)

\( R_b \) is the ratio of beam radiation on tilted surface to that on plane of measurement may be obtained by Eq. (8) [10]

\[ R_b = \frac{\cos(\phi - s) \cos \delta \cos \psi + \sin(\phi - s) \sin \delta}{\cos \phi \cos \delta \cos \psi + \sin \phi \sin \delta} \]  

(8)
When water flows into the storage tank, we should apply energy balance equations including both PCM and water. Sensible and latent heat are represented in Eqs. (9) and (10) [6]

\[ Q_S = m_w c_{p,w} \Delta T + m_{PCM} \left( c_{p,PCM} - L \Delta T_1 + c_{p,PCM} - s \Delta T_2 \right) \]  

(9)

\[ Q_L = m_{PCM} h_{SL} \]  

(10)

The total heat for the storage tank may be obtained by summation of sensible and latent heat as shown in Eq. (11) [6]

\[ Q_{PCM} = Q_S + Q_L \]  

(11)

Where \( Q_{PCM} \) is the total energy stored in PCM minus the energy stored in water. After this, the discharge time of PCM which is the period that hot water is available after sunset can be calculated [6]

\[ t_{PCM} = \frac{Q_{PCM}}{\dot{Q}_w} \]  

(12)

In Eq. (12) \( \dot{Q}_w \) is the total heat stored per unit time in the conventional storage tank and can be given as follows

\[ \dot{Q}_w = \dot{m}_w c_p (T_{PCM m} - T_i) \]  

(13)

Finally \( Q_{net} \) can be calculated as follows

\[ Q_{net} = m_w c_{p,w} (T_{out} - T_{in}) - \dot{Q}_{PCM} \]  

(14)

It is obvious that in a solar system the net energy stored (\( Q_{net} \)) and discharge time of PCM (\( t_{PCM} \)) should be maximized. Design variables in present study are: inner diameter of pipes (\( D \)), area of collectors (\( A_C \)), water mass flow rate (\( \dot{m} \)), tilt of collector (\( s \)) and the mass of PCM (\( M_{PCM} \)). The range of variations of design variables are shown in Table 1 moreover some other constants in the mathematical modeling are shown in Table 2.

<table>
<thead>
<tr>
<th>Table 1: Design variables and their range of variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Variable</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>( D ) (m)</td>
</tr>
<tr>
<td>( A_C ) (m²)</td>
</tr>
<tr>
<td>( \dot{m} ) (kg/s)</td>
</tr>
<tr>
<td>( s ) (deg)</td>
</tr>
<tr>
<td>( M_{PCM} ) (kg)</td>
</tr>
</tbody>
</table>

Transactions of the Canadian Society for Mechanical Engineering, Vol. 34, No. 3–4, 2010
3. MULTI-OBJECTIVE OPTIMIZATION

The concept of Pareto front or set of optimal solutions in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. This means that it is not possible to find a single solution to be superior to all other solutions with respect to all objectives so that changing the vector of design variables in such a Pareto front consisting of these non-dominated solutions could not lead to the improvement of all objectives simultaneously. Consequently, such a change will lead to deteriorating of at least one objective. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space. Such problems can be mathematically defined as:

Find the vector \( \mathbf{X} = [x_1, x_2, \ldots, x_n]^T \) to optimize

\[
F(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_k(\mathbf{X})]^T
\]  

(15)

Subject to \( m \) inequality constraints

\[
g_i(\mathbf{X}) \leq 0, \quad i = 1 \text{ to } m
\]  

(16)

And \( p \) equality constraints

\[
h_j(\mathbf{X}) = 0, \quad j = 1 \text{ to } p
\]  

(17)

Where \( X^* \in \mathbb{R}^n \) is the vector of decision or design variables, and \( F(\mathbf{X}) \in \mathbb{R}^k \) is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

3.1. Definition of Pareto dominance

A vector \( \mathbf{U} = [u_1, u_2, \ldots, u_k] \in \mathbb{R}^k \) is dominant to vector \( \mathbf{V} = [v_1, v_2, \ldots, v_k] \in \mathbb{R}^k \) (denoted by \( \mathbf{U} < \mathbf{V} \)) if and only if \( \forall i \in \{1, 2, \ldots, k\} \), \( u_i \leq v_i \) and \( \exists j \in \{1, 2, \ldots, k\} : u_j < v_j \). In other words, there is at least one \( u_j \) which is smaller than \( v_j \) whilst the remaining \( u \)'s are either smaller or equal to corresponding \( v \)'s.

3.2. Definition of Pareto optimality

A point \( X^* \in \Omega \) ( \( \Omega \) is a feasible region in \( \mathbb{R}^n \) satisfying equations (13) and (14)) is said to be Pareto optimal (minimal) with respect to all \( X \in \Omega \) if and only if \( F(X^*) < F(X) \). Alternatively, it can be readily restated as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting point of PCM (°C)</td>
<td>35</td>
</tr>
<tr>
<td>Melting latent heat of PCM (kJ/kg)</td>
<td>278.84</td>
</tr>
<tr>
<td>Density of PCM (kg/m³)</td>
<td>1522</td>
</tr>
<tr>
<td>Ambient temperature (°C)</td>
<td>25</td>
</tr>
<tr>
<td>Inlet water temperature (°C)</td>
<td>15</td>
</tr>
<tr>
<td>Specific heat of solid (kJ/kg)</td>
<td>1.55</td>
</tr>
<tr>
<td>Specific heat of Liquid (kJ/kg)</td>
<td>2.51</td>
</tr>
</tbody>
</table>

Table 2: Some constants used in mathematical modeling
∀i ∈ \{1,2,\ldots,k\}, ∀X ∈ \Omega = \{X^*\} f_i(X^*) ≤ f_i(X) \land \exists j \in \{1,2,\ldots,k\} : f_j(X^*) < f_j(X). In other words, the solution \(X^*\) is said to be Pareto optimal (minimal) if no other solution can be found to dominate \(X^*\) using the definition of Pareto dominance.

3.3. Definition of a Pareto Set
For a given MOP, a Pareto set \(P^*\) is a set in the decision variable space consisting of all the Pareto optimal vectors \(P^* = \{X ∈ \Omega \mid \nexists X' ∈ \Omega : F(X') < F(X)\}\). In other words, there is no other \(X'\) as a vector of decision variables in \(\Omega\) that dominates any \(X ∈ P^*\).

3.4. Definition of a Pareto front
For a given MOP, the Pareto front \(P^f\) is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set \(P^*\), that is \(P^f = \{F(X) = (f_1(X), f_2(X), \ldots, f_k(X)) : X ∈ P^*\}\). In other words, the Pareto front \(P^f\) is a set of the vectors of objective functions mapped from \(P^*\).

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most of the difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. In this way, the original non-dominated sorting procedure given by Goldberg [2] was the catalyst for several different versions of multi-objective optimization algorithms. In this paper, the premature

Fig. 2. Multi-objective Pareto result for solar system.
convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front using a recently developed algorithm, namely, the $\epsilon$-elimination diversity algorithm by some of authors [11, 12].

4. MULTI-OBJECTIVE OPTIMIZATION OF SOLAR SYSTEM

In order to investigate the optimal performance of the solar thermal energy storage in different conditions, GAs is now employed in a multi-objective optimization procedure. The two conflicting objectives in this study are the net energy ($Q_{\text{net}}$) and discharge time of PCM ($t_{\text{PCM}}$) that are to be simultaneously optimized with respect to the design variables $D$, $A_C$, $\dot{m}$, $s$ and $M_{\text{PCM}}$ (Table 1). The multi-objective optimization problem can be formulated in the following form:

![Optimal variations of discharge time with respect to mass flow rate.](image)

Table 3. The values of objective functions and their associated design variables of the optimum points

<table>
<thead>
<tr>
<th>P</th>
<th>Design Variables</th>
<th>Objective Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D$ (m)</td>
<td>$A_C$ (m$^2$)</td>
</tr>
<tr>
<td>A</td>
<td>0.008</td>
<td>0.66</td>
</tr>
<tr>
<td>B</td>
<td>0.008</td>
<td>0.66</td>
</tr>
<tr>
<td>C</td>
<td>0.008</td>
<td>0.66</td>
</tr>
<tr>
<td>D</td>
<td>0.008</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Fig. 3. Optimal variations of discharge time with respect to mass flow rate.
\begin{align*}
\text{Maximize} & \quad Q_{\text{Net}} = f_1(D, A_C, \dot{m}, s, M_{\text{PCM}}) \\
\text{Maximize} & \quad t_{\text{PCM}} = f_2(D, A_C, \dot{m}, s, M_{\text{PCM}}) \\
\text{Subject to :} & \quad \begin{cases}
0.008 \leq D \leq 0.02 \\
0.25 \leq A_C \leq 1 \\
0.0015 \leq \dot{m} \leq 0.005 \\
10 \leq s \leq 45 \\
5 \leq M_{\text{PCM}} \leq 20
\end{cases}
\end{align*}

The evolutionary process of Pareto multi-objective optimization is accomplished by using the recently developed algorithm, namely, the c-elimination diversity algorithm where a population size of 200 has been chosen in all runs with crossover probability $P_c$ and mutation probability $P_m$ as 0.7 and 0.07, respectively.

Figure 2 depicts the obtained non-dominated optimum design points as a Pareto front of those two objective functions. There are four optimum design points, namely, $A$, $B$, $C$ and $D$ whose corresponding designs variables and objective functions are shown in Table 3. These points clearly demonstrate tradeoffs in objective functions $Q_{\text{net}}$ and $t_{\text{PCM}}$ from which an appropriate design can be compromisingly chosen.

It is clear from Fig. 2 that all the optimum design points in the Pareto front are non-dominated and could be chosen by a designer as optimum solar system. Evidently, choosing a better value for any objective function in the Pareto front would cause a worse value for another.

![Graph](image)

**Fig. 4.** Optimal variations of net energy with respect to mass flow rate.
objective. The corresponding decision variables of the Pareto front shown in Fig. 2 are the best possible design points so that if any other set of decision variables is chosen, the corresponding values of the pair of objectives will locate a point inferior to this Pareto front. Such inferior area in the space of the two objectives is in fact bottom/ left side of Fig. 2.

In Fig. 2, the design points $A$ and $D$ stand for the best $Q_{net}$ and $t_{PCM}$ respectively. Moreover, the design point, $B$ exhibit important optimal design concepts. In fact, optimum design point $B$ obtained in this paper exhibits a decrease in $Q_{net}$ (about 4.77%) in comparison with that of point $A$, whilst its $t_{PCM}$ improves about 20.4% in comparison with that of point $A$.

It is now desired to find a trade-off optimum design points compromising both objective functions. This can be achieved by the method employed in this paper, namely, the mapping method [13, 14]. In this method, the values of objective functions of all non-dominated points are mapped into interval 0 and 1. Using the sum of these values for each non-dominated point, the trade-off point simply is one having the minimum sum of those values. Consequently, optimum design point $C$ is the trade-off points which have been obtained from the mapping method.

There are some interesting design facts which can be used in the design of such solar systems. Fig. 3 demonstrates that the optimal behaviors of $t_{PCM}$ with respect to mass flow rate can be readily represented by

$$t_{PCM} \propto \dot{m}^2$$

(19)

Fig. 5. Optimal variations of discharge time with respect to mass of PCM.
Figure 4 represents the optimal relationship of net energy and mass flow rate in the form of

\[ Q_{\text{net}} \propto \dot{m} \]  

The optimal variations of \( Q_{\text{net}} \) and \( t_{\text{PCM}} \) with respect to \( M_{\text{PCM}} \) are shown in Figs. 5 and 6 respectively. As seen from these figures, from point A to B, \( M_{\text{PCM}} \) varies linearly. These useful relationships that indefeasible between the optimum design variables of a solar system cannot be discovered without the use of multi-objective Pareto optimization process presented in this paper.

5. CONCLUSION

Multi-objective Pareto based on optimization of solar system has been successfully used. Current Pareto optimal solutions display tradeoff information between maximization of net energy stored and discharge time of PCM. Such tradeoff information is very helpful to a higher-level decision-maker in selecting a design with other considerations.

REFERENCES