ABSTRACT
A hybrid approach of combining Taguchi method, principal component analysis and fuzzy logic for the tolerance design of a dual-purpose six-bar mechanism is proposed. The approach is to firstly use the Taguchi orthogonal array to carry out experiments for calculating the S/N ratios of the positional errors to the angular error of the dual-purpose six-bar mechanism. The principal component analysis is then applied to determine the principal components of the S/N ratios, which are transformed via fuzzy logic reasoning into a multiple performance index (MPI) for further analysis of the effect of each control factors on the quality of the mechanism. Through the analysis of response table and diagram, key dimensional tolerances can be classified, which allows the decision of either to tighten the key tolerances to improve mechanism quality or to relax the tolerance of non-key dimensions to reduce manufacturing costs to be made.

COMBINAISON DE LA MÉTHODE TAGUCHI, ANALYSE DU COMPOSANT PRINCIPAL ET LOGIQUE FLOUE, POUR LE DESIGN TOLÉRANT D'UN MÉCANISME À SIX BARRES À DOUBLE FONCTION

RÉSUMÉ
Une approche hybride combinant la méthode Taguchi, analyse du composant principal et logique floue, pour le design tolérant d’un mécanisme à six barres à double fonction, est proposée. L’approche consiste à utiliser premièremment la table orthogonale de Taguchi pour mener des expériences de calcul de ratio S/N des erreurs de position à l’erreur angulaire du mécanisme à six barres à double fonction. L’analyse du composant principal est alors appliquée pour déterminer les composants principaux des ratios, lesquels sont transformés via le raisonnement de la logique floue en un indice de performance multiple pour une analyse plus complète des effets de chaque facteur de contrôle sur la qualité de mécanisme. Par l’analyse de la table de réponse et du diagramme, les tolérances dimensionnelles clés peuvent être classifiées. Ceci permet de décider soit du resserrement des tolérances clés pour améliorer la qualité ou de détendre la tolérance dimensionnelle non-clés pour réduire les coûts de fabrication.
1. INTRODUCTION

Linkages can be classified into three types: function generation, path generation and motion generation mechanisms. When the relationship between the output link and the input link satisfies specific functions, the mechanism is known as a function generation mechanism. If the position of a point on the output link relative to the input link forms a specific path, the mechanism is called a path generation mechanism. If the output link moves in a series of specified motions according to the guide on the input link, the mechanism is known as a motion generation mechanism.

Complex mechanisms are common in engineering applications. These mechanisms are generally a combination of two of the above three types of mechanisms, combined to satisfy requirements of practical engineering applications. One of such mechanisms is shown in Fig. 1, a dual-purpose six-bar linkage [1]. The function relationship between the input and output links is \( y = x^2 \). The locus of the point \( P \) of the mechanism is a straight line. This mechanism possesses both the characteristics of a function generator and path generator and is therefore classed as a dual-purpose six-bar mechanism.

The synthesis of these linkages can be separated into two types, one that passes through specified precise positions and one that is optimised. Whilst the closed loop synthesis of the first type can guarantee that the designed mechanism would pass through the specified points precisely, it is limited by the number of points. The optimization method, on the other hand, results in a mechanism that passes through a series of prescribed points. The positional accuracy of linkages is paramount in the design of mechanisms. Even with the most precise design, when the design is converted into prototypes, there will be inevitable constraints such as manufacturing tolerance, joint clearances as well as the deflection and thermal deformation of links, all of which affect the accuracy of the mechanism. These ultimately affect the performance of the design mechanism.

Expectedly, a tight tolerance would significantly increase the cost of manufacture whereas a large tolerance would result in inevitable assembly problems affecting the overall performance of the system. The key is thus to specify an optimal dimensional tolerance that ensures the system fulfils its specified requirements. Inadequate design of tolerance on link lengths and

Fig. 1. Dual-purpose six-bar mechanism.
clearance in joints may cause mechanical error of appreciable magnitude as has been documented in numerous papers on the analysis of linkages over the past decades. In earlier studies, Garrett and Hall [2] investigated the effect of tolerance and clearance in linkage design. A few researchers [3–6] then introduced variations to linkage synthesis and presented a mechanism synthesis method accounting for manufacturing tolerances and cost incurred by function generation problems. Dhande and Chakraborty [3] proposed a stochastic model, also known as the equivalent linkage model, for analysing the mechanical error of a four-bar function generator by considering the net effect of tolerance and clearance on the length of the equivalent link. Using the equivalent linkage model, Mallik and Dhande [7] performed the analysis and synthesis of four-bar path generators. Lee and Gilmore [8] proposed an effective link length model to generalize the equivalent linkage concept and carried out a sensitivity study on infinitesimal motion.

The sensitivity of the mechanism is defined as the ratio of the change of a given output variable to the change in a design parameter. Faik and Erman [9] introduced a non-dimensional sensitivity coefficient for individual links of a mechanism rather than using a single value for the entire system. The work was later extended to the sensitivity synthesis of four-bar linkages with three and four prescribed precision positions [10]. It is known that a Jacobian matrix can describe the partial relationship between the output tolerance and the dimensional tolerance of a mechanism. Wu and Lankarani [11] used the determinant of a constrained Jacobian matrix as the sensitivity indicator of an entire mechanism. Ting and Long [12] presented a general theory to determine the sensitivity of tolerances to the performance quality of mechanisms and also a technique to identify robust designs. They demonstrated the effect of tolerance specification on performance quality and showed that performance quality can be significantly improved by tightening key tolerance while relaxing less major ones. Zhu and Ting [13] used the theory of performance sensitivity distribution to study the sensitivity of a system to variations. They defined a tolerance box as the contracting circumscribing box of the design sensitivity ellipsoid of a mechanism. Caro et al. [14] proposed an efficient tolerance synthesis method that computed the optimal tolerance box of a selected robust manipulator by finding the largest tolerance box of a mechanism.

In 1980, Taguchi proposed the concepts of designing parameters and tolerances. The philosophy of the Taguchi method [15–17] is to achieve a robust engineering design through optimizing design parameters against sensitivity to parameter variations. This is in contrast to the traditional method of controlling the source of the variation at all cost. Despite the successful application of the Taguchi method to improving manufacturing processes and to the development of products, its application on the synthesis of mechanism is only reported by Kota and Chiou [18] in 1993. The design parameters were link dimensions and these were assigned as the control factors with their corresponding tolerances as the noise factors. Using the orthogonal array of the Taguchi method, the optimum dimension of the path generator of a mechanism was synthesized.

Most published Taguchi applications to date are related to the optimization of a single performance characteristic. Also of much interest is the handling of the more demanding multiple performance characteristics (MPCs) [19–20]. Principal component analysis was proposed by Pearson and developed into a computational method by Hotelling [21]. In research studies, it is common to encounter conditions where MPCs are possibly inter-related. The current pressing need is to reduce the number of variables and to make them independent or, at the least, having linear inter-relationships, in other words, making them the so called potential variables. Clearly, using fewer potential variables or components to effectively represent the
complex inter-relationship between parameters in a structure is extremely cost effective. In this case, the principal component analysis method is appropriate to achieve such an objective.

When optimizing a process or product with MPCs, the objective is to determine the appropriate design parameters that will simultaneously optimize all the quality characteristics of interest to the designer. The more frequently used approach is to assign a weighting for each response. The difficulty, however, is determining the weighting for each response in an actual case study. The primary ‘weighting’ method is using engineering judgment together with past experiences to optimize MPCs [19]. The consequent results often include some uncertainties in the decision-making process. Using fuzzy logic [22–23], the MPCs can be easily dealt with by setting up a reasoning procedure for each performance characteristic and transforming them into a single value termed as the multiple performance index (MPI).

This paper uses the Taguchi orthogonal array to conduct the experiments for calculating the S/N ratios of the positional and angular errors of a dual-purpose six-bar mechanism. The principal component analysis method is then applied to determine the principal component of the S/N ratios to the quality characteristics. The principal components are then transformed into a MPI by means of fuzzy logic reasoning for analysing the effect of the design parameters on the quality of the six-bar mechanism. Upon understanding the effect of each parameter on the MPI, decisions are then made to either tighten the tolerances to improve quality or to increase the tolerances to reduce cost.

2. TAGUCHI METHOD

All machines or set-ups are classified as engineering systems according to the Taguchi method. As shown in Fig. 2, an engineering system generally consists of four sections: signal factor, control factor, noise factor and output response. Signal factor is the input from the user to the system for specified output response. If the system’s output response changes with the input signal, the system is considered to possess dynamic characteristics according to the Taguchi method. Parameters that are easy or inexpensive to control are usually chosen as the
control factors while those that are difficult or expensive to control are assigned as noise factors.

Taguchi method uses orthogonal array to execute experiments and to analyse results. Using orthogonal array can substantially reduce the time and cost of developing a new product or technique and thereby increase the competitiveness of the product in the open market. Taking the L_{12} (2^{11}) orthogonal array as an example, the initially required 2^{11}=2,048 sets of experiments can be significantly reduced to 12 sets while achieving similar results to a full factorial experimental set-up. Moreover, interaction amongst factors could be evenly distributed to each column, ensuring the effect of interaction is minimized. Orthogonal arrays consist of inner and outer columns, the former assigned with control factors while the latter with input signal and noise factors. The principle behind the Taguchi method is to subject the design parameters to the tests of the noise factors to obtain optimised control factors that are effective in combating the influence of the noise factors acting on the product quality. This ensures the robustness of the system.

2.1. Ideal function, input signal and output response

The dual-purpose six-bar mechanism studied herein is shown in Fig. 3. The mechanism is an amalgam of a function and a path generator. During its motion, the output link, in this case output 1, will pass through precise angular positions while the coupler point (C), output 2, will pass through a series of points. Its quality characteristic is ‘the smaller the errors are, the better the performance is’ and this applies to the angle of the output link and the position of the coupler point. When the angle of the input link is known, the position of all links in the six-bar linkage can be derived using the vector loop method as follows:

$$\theta_2 = 2 \tan^{-1} \left( \frac{-F_I \pm \sqrt{E_I^2 + F_I^2 - G_I^2}}{G_I - E_I} \right)$$

(1)

![Fig. 3. Vector loop diagram of the dual-purpose six-bar mechanism.](image)
\[ \theta_3 = \tan^{-1} \left( \frac{r_1 \sin \theta_1 + r_2 \sin \theta_4 + r_4 \sin \theta_2}{r_1 \cos \theta_1 + r_2 \cos \theta_4 + r_4 \cos \theta_2} \right) \]  

(2)

\[ \theta_8 = 2 \tan^{-1} \left( \frac{-F_2 \pm \sqrt{F_2^2 + G_2^2}}{G_2 - F_2} \right) \]  

(3)

\[ \theta_7 = \tan^{-1} \left( \frac{-r_1 \sin \theta_1 - r_3 \sin \theta_5 - r_8 \sin \theta_9 - r_9 \sin \theta_9}{-r_1 \cos \theta_1 - r_3 \cos \theta_5 - r_8 \cos \theta_8 - r_9 \cos \theta_9} \right) \]  

(4)

where \( \theta_i \): angular position of the vector \( r_i \) with respect to the horizontal \((X_o)\)

\[ E_1 = 2r_2(r_1 \cos \theta_1 + r_4 \cos \theta_4) \]

\[ F_1 = 2r_2(r_1 \sin \theta_1 + r_4 \sin \theta_4) \]

\[ G_1 = r_1^2 + r_2^2 + r_3^2 + r_4^2 + 2r_1r_4 \cos(\theta_1 - \theta_4) \]

\[ E_2 = 2r_8(r_1 \cos \theta_1 + r_5 \cos \theta_5 + r_9 \cos \theta_9) \]

\[ F_2 = 2r_8(r_1 \sin \theta_1 + r_5 \sin \theta_5 + r_9 \sin \theta_9) \]

\[ G_2 = r_1^2 + r_5^2 - r_7^2 + r_8^2 + r_9^2 + 2r_1r_5 \cos(\theta_1 - \theta_5) + 2r_1r_9 \cos(\theta_1 - \theta_9) + 2r_5r_9 \cos(\theta_5 - \theta_9) \]

\[ \theta_2 = \theta_5 + \alpha \]

At this point, the coordinates of coupler point \((C)\) can be expressed as

\[ (x_c, y_c) = (x_1 + r_1 \cos \theta_1 + r_5 \cos \theta_5, y_1 + r_1 \sin \theta_1 + r_5 \sin \theta_5) \]  

(5)

For the six-bar mechanism, the angular displacement of the output link of the function generator \( \psi_s \) \( (= \theta_8) \) deviates from the ideal value \( \psi_i \) with the difference \( \varepsilon_s = \psi_s - \psi_i \) known as the structural error. The combined angular displacement error of the links due to manufacturing tolerance and clearances at the joints is known as mechanical error and is defined as \( \varepsilon_m = \psi_m - \psi_s \).
Therefore the error between the ideal and actual mechanism, i.e. the ideal function model for the quality characteristic prediction is given as

\[ y_1 = \varepsilon = \varepsilon_s + \varepsilon_m = (\psi_s - \psi_i) + (\psi_m - \psi_s) = \psi_m - \psi_i \] (6)

As the structural error is much less than the mechanical error \[24\], Eq. (6) can be simplified as \( y_1 = \varepsilon \equiv \varepsilon_m \), whose characteristic is described as ‘smaller-the-better’, i.e. with an ideal value of 0.

For a dual-purpose six-bar mechanism, the positional error of the coupler point has the characteristic of ‘the-smaller-the-better’ and therefore has an ideal value of 0. Therefore the ideal function model can be expressed as

\[ y_2 = |E_x| \] (7)

\[ y_3 = |E_y| \] (8)

where \( E_x \) and \( E_y \) are correspondingly, the error.

The input signal \( \varphi \) comes from the angular position of input link. For every \( 5^\circ \) made by the input link, a total of five angles \( (M_1, M_2, \ldots, M_5) \) \[1\] are recorded as the input signal in order to compute the position of the coupler point and the angle of the output link of the six-bar mechanism. The difference between the actual position and the ideal position is then taken as the output response. This difference has the characteristics of ‘the-smaller-the-better’ and therefore has an ideal value of 0. The S/N expression can be expressed as

\[ S/N = \eta = 10\log_{10} \frac{1}{n} \sum_{j=1}^{n} \frac{1}{y_{ij}^2} \], \( i = 1, 2, 3 \) (9)

where \( y_{1i}, y_{2i} \) and \( y_{3i} \) are angular error(mechanical error) , \( X \) and \( Y \) positional errors of the mechanism, respectively; \( n = 5 \) is the number of the output responses relative to input signals (five input angles).

The Taguchi method is first applied to determine three S/N ratios of the \( X \) and \( Y \) position of the coupler and the angle of the output link. The principal component analysis method is then applied to calculate the main contributor of the S/N ratio to the quality characteristics. The S/N ratio is subsequently transformed to a MPI according to the contribution level to further analyse the effects of the control factors on the quality of the dual-purpose six-bar mechanism.

2.2. Control factors and levels

Control factors are parameters in a mechanism that can be easily or economically controlled. By means of vector loop method, the position of each link of the dual-purpose six-bar mechanism can be determined. The associated design parameters are thus the tolerances of the links, \( r_1, r_2, r_3, r_4, r_5, r_7, r_8, r_9 \) as well as angles \( \theta_4, \theta_9 \) and \( \alpha \). The parameters of this mechanism are listed in Table 1 as the nominal values \[1\]. The net change in the equivalent link’s length due to manufacturing tolerance is taken as the equivalent link length tolerance, \( \sigma_l = 0.0025 \text{ mm} \), and the angular tolerance of \( \sigma_\theta = 0.01 \). In 2-level tolerance design experiments, the factor levels are established as \[17\]
\[ Level 1 = \text{Nominal} - \sigma \]  

\[ Level 2 = \text{Nominal} + \sigma \]

where \( \sigma \) is equal to \( \sigma_l \) or \( \sigma_a \).

### 2.3 Noise factors and levels

Noise factors are unique operating parameters that are not easy or immensely costly to control. For a six-bar mechanism, it is recognised that there are various factors that are difficult to control. These include the link deformations due to link loading and temperature changes. Another is the operating temperature that will result in thermal expansion or compression of the link lengths. As the aim of this paper is to obtain the sensitivity of the dimensions of the links, these noise factors will not be considered herein. This study uses a L\(_{12}\) orthogonal array to conduct the experiments and the afore-defined input signal and control factors are assigned into the array as shown in Table 2.

### 3. PRINCIPAL COMPONENT ANALYSIS

Principal component analysis [21] is a technique that provides a way to explore multivariate data. The original initial variables are transformed into another dimensional set of uncorrelated variables, e.g., the principal components are transformed by calculating the eigenvectors of the covariance matrix of the original inputs. The transformed variables are ranked according to their variance, reflecting a decreasing importance, in order to capture the whole information content of the original dataset.

Although \( p \) components are required to reproduce the total system variability, often much of this variability can be accounted for by a small \( k \) of the principal components. The \( k \) principal components can then replace the initial \( p \) variables, and the original dataset, consisting of \( n \) measurements on \( p \) variables, is reduced to a dataset consisting of \( n \) measurements on \( k \) principal components.
Suppose \( X = (X_1, X_2, \ldots, X_p)^T \) contains the covariance matrix \( \sum \), with eigenvalue-eigenvector pairs \((\lambda_1, V_1), (\lambda_2, V_2), \ldots, (\lambda_p, V_p)\) where \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p \geq 0 \). The \( i \)th principal component is given by

\[
Y_i = V_{i1}X_1 + V_{i2}X_2 + \ldots + V_{ip}X_p \quad (i = 1, 2, \ldots, p)
\]

With \( \text{Var}(Y_i) = V_i^T \sum V_i = \lambda_i \) and \( \text{Cov}(Y_i, Y_k) = V_i^T \sum V_k = 0, i \neq k \). The total system variance is given by

\[
\sum_{i=1}^{p} \text{Var}(X_i) = \sum_{i=1}^{p} \text{Var}(Y_i) = \sum_{i=1}^{p} \lambda_i
\]

The proportion of total variance explained by the \( i \)th principal component is defined as its explanatory power. The explanatory power of the \( i \)th principal component is \( \frac{\lambda_i}{\sum \lambda_i} \).

Table 2. Experimental layout and S/N ratio.

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\( \bar{X}_i(k) = \frac{x_i(k) - \bar{x}_i}{s} \quad i = 1, 2, 3 \)

where \( \bar{x}_i(k) \) denotes the value after standardization for the \( k \)th test (\( i = 1 \) for error in the \( X \) direction; \( i = 2 \) for error in the \( Y \) direction; \( i = 3 \) for the angular error), \( x_i(k) \) the original
experimental response, 
\[ \bar{x}_i = \frac{\sum_{k=1}^{n} x_i(k)}{n} \] the average value for the original experimental response, 
\[ s = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} [x_i(k) - \bar{x}]^2} \] the standard deviation before standardization.

4. FUZZY LOGIC

Fuzzy logic is a mathematical theory of inexact reasoning that allows modeling of the reasoning process of human in linguistic terms [25]. It is very suitable in defining the relationship between system inputs and desired outputs. A fuzzy logic system comprises a fuzzifier, membership functions, fuzzy rules, an inference engine, and a defuzzifier. The basic process sequence is such that the fuzzifier first uses membership functions to convert inputs into fuzzy sets. Then the inference engine performs a fuzzy reasoning on fuzzy rules to generate fuzzy values, which are subsequently converted into crisp outputs by the defuzzifier. The flow structure chart of the integrated fuzzy logic controller, the Taguchi methods and the principal component analysis used in the study is shown as Fig. 2.

In the following section, the concept of fuzzy reasoning is described briefly based on the two-input-one-output fuzzy logic system. For this system, the fuzzy rule base consists of a group of if-then control rules with two inputs, \( x_1 \) and \( x_2 \), and one output \( y \), i.e.

Rule 1: if \( x_1 \) is \( A_1 \) and \( x_2 \) is \( B_1 \) then \( y \) is \( C_1 \) else
Rule 2: if \( x_1 \) is \( A_2 \) and \( x_2 \) is \( B_2 \) then \( y \) is \( C_2 \) else
………………………………………………………
Rule m: if \( x_1 \) is \( A_m \) and \( x_2 \) is \( B_m \) then \( y \) is \( C_m \).

\( A_i, B_i, \) and \( C_i \) are fuzzy subsets defined by the corresponding membership function, i.e., \( \mu_{A_i}, \mu_{B_i}, \) and \( \mu_{C_i}. \) In this paper, five and three fuzzy subsets are assigned to the two inputs as shown in Figs. 4(a) and (b) respectively. Five fuzzy subsets are assigned to the output as shown in

![Fig. 4](image-url)

Fig. 4. Membership functions, (a) membership function of input 1, (b) membership function of input 2, (c) membership function of output.
Fig. 4(c). Various degree of membership to the fuzzy sets is calculated based on the values of $x_1$ and $x_2$. Fifteen fuzzy rules listed in Table 3 are established directly based on the premise that the larger is the value, the better is the performance. By taking the min-max compositional operation, the fuzzy reasoning of these rules yields a fuzzy output. Based on the Mamdani implication method of inference reasoning for a set of disjunctive rules, the aggregated output for the $m$ rules is

$$
\mu_{Co}(y) = \left[ \mu_{A_1}(x_1) \land \mu_{B_1}(x_2) \right] \lor \left[ \mu_{A_2}(x_1) \land \mu_{B_2}(x_2) \right] \lor \ldots \lor \left[ \mu_{A_m}(x_1) \land \mu_{B_m}(x_2) \right]
$$

where $\land$ is the minimum operation and $\lor$ is the maximum operation. The above equation is illustrated in Fig. 5. The graph represents the fuzzy reasoning process for two rules with two input variables that uses triangular-shape membership functions. Using a defuzzification method, fuzzy values can be combined into one single crisp output value, as shown in Fig. 5. The center of gravity, one of the most popular methods for defuzzifying fuzzy output functions, is employed in the study. The formula to find the centroid of the combined outputs, $y_o$, is given by:

$$
y_o = \frac{\int y \mu_{Co}(y) dy}{\int \mu_{Co}(y) dy}
$$

In this paper, the non-fuzzy value $y_o$ is called the MPI and the premise is the larger is the MPI, the better is the performance.

<table>
<thead>
<tr>
<th>Multiple performance index (MPI)</th>
<th>Input 1</th>
<th>Input 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XS</td>
<td>S</td>
</tr>
<tr>
<td>Input 2</td>
<td>XS</td>
<td>XS</td>
</tr>
<tr>
<td>M</td>
<td>XS</td>
<td>S</td>
</tr>
<tr>
<td>L</td>
<td>S</td>
<td>M</td>
</tr>
</tbody>
</table>

Fig. 5. Fuzzy logic reasoning process.
According to the nominal values of the control factors in Table 1, the positional errors of the coupler point in the $X$ and $Y$ directions, $g_1$ and $g_2$, as well as the angular error of the output link of the dual-purpose six-bar mechanism, $g_3$, can be analysed. These are then used to evaluate the S/N ratios. Table 2 shows the computer simulated results. The standardized S/N ratios are shown in Table 4.

The values are then input into a computational software to calculate the correlation matrix. This process is to determine if the principal component analysis method is applicable to this set of variables. If the variables do not possess any correlation, then the returned correlation value is 0 and the analysis is terminated. From Table 5 it can be seen that the correlation of the positional error of the coupler in the $X$ and $Y$ directions is as high as 0.954, suggesting that if the error in the $X$ direction is improved, the error in the $Y$ direction would simultaneously be improved. There is also a significant correlation between the positional error of the coupler in the $X$ direction and the angle of the output link with a corresponding correlation value of 0.967.

Using the correlation matrix, the eigenvalues and eigenvectors, representing the magnitudes and directions of the principal components, respectively, can be computed. These are shown in Table 6. The eigenvectors are multiplied by the standardized values as shown in the equation below:

$$[Y_1] = \begin{bmatrix} 0.5780 & 0.5757 & 0.5784 \\ 0.4568 & -0.8155 & 0.3553 \\ 0.6762 & 0.0588 & -0.7343 \end{bmatrix} [\eta_{1s} \eta_{2s} \eta_{3s}]$$

<table>
<thead>
<tr>
<th>L12</th>
<th>$\eta_{1s}$</th>
<th>$\eta_{2s}$</th>
<th>$\eta_{3s}$</th>
<th>First principal component $Y_1$</th>
<th>Second principal component $Y_2$</th>
<th>Third principal component $Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8947</td>
<td>1.3922</td>
<td>0.7107</td>
<td>1.7297</td>
<td>-0.4741</td>
<td>0.1650</td>
</tr>
<tr>
<td>2</td>
<td>0.9497</td>
<td>0.6194</td>
<td>0.7780</td>
<td>1.3555</td>
<td>0.2051</td>
<td>0.1073</td>
</tr>
<tr>
<td>3</td>
<td>1.3359</td>
<td>0.8407</td>
<td>1.0597</td>
<td>1.8690</td>
<td>0.3012</td>
<td>0.1746</td>
</tr>
<tr>
<td>4</td>
<td>-0.9537</td>
<td>-0.6948</td>
<td>-1.1325</td>
<td>-1.6063</td>
<td>-0.2715</td>
<td>0.1458</td>
</tr>
<tr>
<td>5</td>
<td>0.6304</td>
<td>0.9140</td>
<td>0.9767</td>
<td>1.4554</td>
<td>-0.1104</td>
<td>-0.2372</td>
</tr>
<tr>
<td>6</td>
<td>-0.7528</td>
<td>-0.9446</td>
<td>-0.9559</td>
<td>-1.5318</td>
<td>0.0868</td>
<td>0.1373</td>
</tr>
<tr>
<td>7</td>
<td>0.9741</td>
<td>0.7627</td>
<td>0.7911</td>
<td>1.4597</td>
<td>0.1040</td>
<td>0.1226</td>
</tr>
<tr>
<td>8</td>
<td>-0.9103</td>
<td>-1.0422</td>
<td>-1.0261</td>
<td>-1.7196</td>
<td>0.0695</td>
<td>0.0766</td>
</tr>
<tr>
<td>9</td>
<td>-1.1443</td>
<td>-1.0606</td>
<td>-0.8079</td>
<td>-1.7393</td>
<td>0.0552</td>
<td>-0.2429</td>
</tr>
<tr>
<td>10</td>
<td>0.8347</td>
<td>1.0941</td>
<td>1.3059</td>
<td>1.8677</td>
<td>-0.0469</td>
<td>-0.3302</td>
</tr>
<tr>
<td>11</td>
<td>-0.6747</td>
<td>-0.9071</td>
<td>-0.5904</td>
<td>-1.2537</td>
<td>0.2218</td>
<td>-0.0761</td>
</tr>
<tr>
<td>12</td>
<td>-1.1834</td>
<td>-0.9738</td>
<td>-1.1095</td>
<td>-1.8863</td>
<td>-0.1406</td>
<td>-0.0428</td>
</tr>
</tbody>
</table>

Table 4. Standardized results and Principal Components.

<table>
<thead>
<tr>
<th>Error in the $X$ direction</th>
<th>Error in the $Y$ direction</th>
<th>Angular Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.954</td>
<td>0.967</td>
</tr>
<tr>
<td>0.954</td>
<td>1.000</td>
<td>0.956</td>
</tr>
<tr>
<td>0.967</td>
<td>0.956</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5. Correlation matrix.
The principal components of the quality characteristics, i.e. $Y_1, Y_2, Y_3$, can be determined as shown in Table 4.

The next step is to normalize the three principal components to a value between 0 and 1. Then, the membership functions formed by the fuzzy logic input from Figs. 4(a) and (b), together with the output membership functions in Fig. 4(c) and the fuzzy logic rules in Table 3 are applied into the 2-step fuzzy method to transform the three normalized principal components into a MPI. The flow chart is shown in Fig. 6. The first step is to obtain MPI-1 from using the normalized second and third principal components as input 1 and input 2, respectively. Fig. 7 shows a screenshot of the graphics user interface of the toolbox. The second step is to apply the fuzzy logic rules with the first normalized main component as input 1 and MPI-1 as input 2 to obtain MPI. Table 7 list the indices evaluated from the 2-step fuzzy reasoning method in the Matlab Fuzzy Tool Box.

Table 8 shows the response of the control factors to the MPIs and Fig. 8 the corresponding response diagram. From the ‘max-min’ range evaluation, the effect of each control factor on the MPI can be determined. From Table 8 and Fig. 8, it is clear that control factor E has the most significant effect on the MPI, followed by the factors C, J, B, G and A, all of which are classed as key dimensions. The tolerance of these parameters should be tightened in order to improve the quality of the mechanism. Alternatively, to reduce manufacturing cost, the tolerance of non-key parameters such as control factors K and J, i.e. $a$ and $\theta_4$, should be relaxed.

The analysis of variance is fundamentally similar to the analysis of the max-min range in the variation response table. The main difference being the former can separate the total variability of the MPIs, which are measured by taking the sum of the squared deviations from the mean of the MPIs, and dividing them into contributions by each of the control factors and the experimental error. The max-min range method, on the other hand, displays the effect of the entire range, including those caused by experimental error. From the result of the analysis of variance shown in Table 9, the variations, $V$, caused by each control factor on the MPI as well as the effect of the control factors on the quality characteristic variation can be observed. The main control factors that can effectively reduce the variations and contribute to the MPI are

Table 6. Eigenvalues and eigenvectors.

<table>
<thead>
<tr>
<th>Principal component</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
<td>2.9180</td>
<td>0.0491</td>
<td>0.0329</td>
</tr>
<tr>
<td>Error in the $X$ direction</td>
<td>0.5780</td>
<td>0.4568</td>
<td>0.6762</td>
</tr>
<tr>
<td>Error in the $Y$ direction</td>
<td>0.5757</td>
<td>-0.8155</td>
<td>0.0588</td>
</tr>
<tr>
<td>Angular error</td>
<td>0.5784</td>
<td>0.3553</td>
<td>-0.7343</td>
</tr>
</tbody>
</table>

Fig. 6. Two-step fuzzy logic reasoning.
Table 7. Normalized principal component and multiple performance index.

<table>
<thead>
<tr>
<th>L12</th>
<th>First principal component $Y_1$</th>
<th>Second principal component $Y_2$</th>
<th>Third principal component $Y_3$</th>
<th>MPI-1</th>
<th>MPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9629</td>
<td>0.0000</td>
<td>0.9810</td>
<td>0.3000</td>
<td>0.777</td>
</tr>
<tr>
<td>2</td>
<td>0.8632</td>
<td>0.8760</td>
<td>0.8668</td>
<td>0.8680</td>
<td>0.866</td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.8940</td>
<td>0.894</td>
</tr>
<tr>
<td>4</td>
<td>0.0746</td>
<td>0.2614</td>
<td>0.9431</td>
<td>0.4630</td>
<td>0.109</td>
</tr>
<tr>
<td>5</td>
<td>0.8899</td>
<td>0.4692</td>
<td>0.1842</td>
<td>0.3290</td>
<td>0.774</td>
</tr>
<tr>
<td>6</td>
<td>0.0944</td>
<td>0.7235</td>
<td>0.9261</td>
<td>0.8910</td>
<td>0.297</td>
</tr>
<tr>
<td>7</td>
<td>0.8910</td>
<td>0.7457</td>
<td>0.8970</td>
<td>0.8840</td>
<td>0.882</td>
</tr>
<tr>
<td>8</td>
<td>0.0444</td>
<td>0.7012</td>
<td>0.8059</td>
<td>0.8310</td>
<td>0.276</td>
</tr>
<tr>
<td>9</td>
<td>0.0392</td>
<td>0.6827</td>
<td>0.1729</td>
<td>0.5210</td>
<td>0.122</td>
</tr>
<tr>
<td>10</td>
<td>0.9996</td>
<td>0.5510</td>
<td>0.0000</td>
<td>0.3590</td>
<td>0.805</td>
</tr>
<tr>
<td>11</td>
<td>0.1685</td>
<td>0.8975</td>
<td>0.5034</td>
<td>0.8910</td>
<td>0.371</td>
</tr>
<tr>
<td>12</td>
<td>0.0000</td>
<td>0.4302</td>
<td>0.5694</td>
<td>0.4690</td>
<td>0.108</td>
</tr>
</tbody>
</table>

Fig. 7. The 15-rule fuzzy logic reasoning to compute MPI-1 of the second experiment.
identified in descending order as \(E(19.91\%), \ C(16.80\%), \ J(15.42\%), \ B(12.39\%), \ G(9.83\%)\) and \(A(9.00\%).\)

### 6. CONCLUSION

A hybrid approach of the Taguchi method, the principal component analysis method and fuzzy logic has been proposed for the tolerance design of a dual-purpose six bar mechanism. The Taguchi orthogonal array has been applied to carry out experiments for calculating the S/N ratios of the positional and angular errors of the mechanism. By using the principle component analysis method, the principal components of the S/N ratios were determined. These were
subsequently transformed into a MPI by means of fuzzy logic reasoning to further analyse the
effect of each control factors on the quality of the mechanism. Through the analysis of response
figures and tables, key dimensional tolerances were identified as E, C, J, B, G and A as having
the most significant effect on the quality of the mechanism. Alternatively, the non-key
dimensions, namely, K ($\alpha$) and I ($\theta_1$), should be relaxed to reduce manufacturing costs. This
approach can be applied to other product designs or for improving process variables.

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