RAY-TRACING ANALYSIS OF A TWO-STAGE SOLAR CONCENTRATOR

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ABSTRACT
A ray-tracing analysis was conducted on a 2-stage solar concentrator made of two parabolic mirrors created by Lunenburg Industrial Foundry & Engineering (LIFE). The effects of the secondary mirror’s focal length, the distance between the secondary mirror and the target, and the misalignment with the sun were studied. The focal length of the secondary mirror determines the maximum local solar energy flux $\Phi$ that can be achieve on the target. For the optimal focal length of 157.9°, a maximum $\Phi = 1.2 \times 10^4$ MW/m$^2$ was achieve compare to $\Phi = 1680$ MW/m$^2$ for the initial LIFE’s focal length of 158.8125°. The concentrator concentrates all the incident energy from the sun on the target, and that independently of the secondary mirror’s focal length (within the range studied), as long as the target position is within an 11 cm zone. Small misalignments in the order of $\pm 0.2^\circ$ would bring the concentration efficiency to zero.

Keywords: ray-tracing analysis; solar energy; solar concentrator; parabolic troughs; solar allignment.

ANALYSE PAR TRACÉ DE RAYONS D'UN CONCENTRATEUR SOLAIRE À 2 MIROIRS

RÉSUMÉ
Une analyse par tracé de rayons a été effectuée sur un concentrateur solaire créé par Lunenburg Industrial Foundry & Engineering (LIFE) et construit à partir de 2 miroirs paraboliques. Les effets de la longueur focale du miroir secondaire, de la distance entre le miroir secondaire et le cible, et l’alignement du concentrateur avec le Soleil ont été étudiés. La longueur focale du miroir secondaire est le facteur principal permettant de déterminer le flux local d’énergie solaire maximum $\Phi$ pouvant frapper la cible. Pour la longueur focale optimale de 157.9°, un $\Phi$ maximum de $1.2 \times 10^4$ MW/m$^2$ a été calculé en comparaison avec un le $\Phi$ obtenu de 1680 MW/m$^2$ pour la longueur focale initiale utilisée par LIFE qui était de 158.8125°. Également, le concentrateur concentre la totalité de l’énergie incidente provenant du Soleil sur la cible choisie, et ce, indépendamment de la longueur focale du miroir secondaire; an autant que la cible se trouve à l’intérieur d’une zone longue de 11 cm. Un mauvais alignement de seulement $\pm 0.2^\circ$ rend l’efficacité du concentrateur nulle.

Mots-clés : analyse par tracé de rayons; énergie solaire; concentrateur solaire; miroirs paraboliques; alignement solaire.
1. INTRODUCTION

With threats of global warming and increased energy demand and cost, the use of renewable energies is becoming more and more popular [1]. Among the various types of renewable energies, solar energy is bound to become a more important source of energy, both thermal and electric, in the near future.

Three types of solar concentrators are currently used to transform large amount of solar energy into thermal energy; they are parabolic dish solar concentrator, parabolic trough reflectors and solar tower [2]. The first two systems will typically achieve temperatures in the 350°C to 400°C range; while the solar tower can achieved temperature as high as 1000°C, but requires a large array of computer controlled mirrors making it the most expensive of the three systems [3].

In recent years, various original designs for 2-stage solar concentrator were developed in order to achieve better heat and electric power generation, sometime approaching the thermodynamic limit [4]. One configuration in particular uses parabolic mirrors to concentrate the sun’s energy on a thermoelectric device [5]; this configuration was developed to achieve high solar flux at the focal area [6].

Lunenburg Industrial Foundry and Engineering (LIFE), a company situated in the town of Lunenburg, Nova Scotia, came up with a new geometry for solar concentrators in an effort to use solar energy to melt metal in their foundry, reducing at the same time their use of fossil fuel. They have since obtained patents for their new geometry [7]. Their system, properly optimized, can achieved concentrated temperatures above 1000°C for a fraction of the price needed to build solar towers. Also, by its simplicity, the solar rays can be concentrated to a system situated on or near the ground, ideal situation to use the concentrator to melt metal.

In 2008, they asked the Laboratory of Applied Multiphase Thermal Engineering (LAMTE) to perform a series of analysis: thermal, structural and optical, on their concentrator. The ray-tracing method is one of the tools that can be used to perform performance studies of solar concentrator. It has been used lately to study stationary solar concentrators [8] and concentrator using both parabolic and hyperbolic mirrors [9]. However, ray-tracing analysis had never been employed to study the optical properties of such 2-stage concentrator.

This paper presents the results of the optical analysis in an effort to answer the following questions:

• Can a point focus be achieved in such a 2-stage solar concentrator? If not, what is the shape of the focal area obtained?

Nomenclature

- \( d \): Target horizontal displacement from origin (m)
- \( \vec{I} \): Incident ray
- \( \vec{n} \): Normal vector
- \( P \): Power
- \( \vec{R} \): Reflected ray

Greek letters

- \( \eta \): Concentration efficiency

\( \Phi \): Maximum point solar energy flux (W/m²)

\( \theta \): Reflection angle (°)

Subscripts

- \( I \): Incident ray
- \( R \): Reflected ray
- \( \text{tot} \): Total
- \( x \): in the \( x \) direction
- \( y \): in the \( y \) direction
- \( z \): in the \( z \) direction

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• What is the effect of the mirrors’ focal length and the distance between the mirrors on the focal area?
• How sensitive is the position of the focal area to the misalignment of the concentrator to the Sun?

First, the geometry of the novel 2-stage solar concentrator will be described. Next, the method used to perform the ray-tracing analysis will be introduced clearly. The results will finally be presented and discussed: the effects of the secondary mirror’s focal length, the distance between the secondary mirror and the target, and the alignment of the concentrator with the sun will be examined.

2. 2-STAGE SOLAR CONCENTRATOR GEOMETRY

LIFE set out to create a solar concentrator that would meet the following design and fabrication criteria:
• In order to melt metal, a spot temperature of at least a 1000 to 1200°C should be attainable;
• For economic reasons, the overall design should be inexpensive to build compared to the aforementioned three types of solar concentrator.

Since none of the existing solar concentrator systems could achieve such high temperatures for a small price, LIFE came up with their own design for a two-stage solar concentrator. Fig. 1 shows a CAD drawing of this concentrator, a 3 dimensional view as well as views from the side, front and top.

In this two-stage solar concentrator, the primary mirror as the shape of an off-axis parabola having a horizontal symmetry axis. Sun rays hitting the primary mirror are reflected toward the secondary mirror (Fig. 2a); this first reflection focuses the rays on a horizontal focal line. The secondary mirror is a symmetrical parabola having a vertical symmetry axis. Rays coming from the primary mirror are reflected by the secondary and focused in the vertical direction toward the receiver positioned at or near the focal area (Fig. 2b). The concentrated solar energy is used, or transformed, or stored, at or near the receiver plane (Fig. 2c).

Being parabolic, these mirrors are easier and cheaper to manufacture than a dish; LIFE currently uses polished stainless steel sheets as the reflective surfaces of the parabolic mirror, they achieve the parabolic curvature of the mirror by inserting the pieces of sheet metal into a casing properly manufactured to give the right parabolic shape. A series of two reflections will concentrate the sun rays onto a small region, the size of which is to be determined through the ray-tracing analysis. To melt metal, the solar collector is put at the focal area of the system in order to achieve the maximum spot temperature.

The frame of LIFE’s first full size prototype was built of aluminum, previous small scale models were made of a mix of wood and steel. The dimensions of the 2-stage solar concentrator on which the ray-tracing analysis was performed are presented on Table 1. Fig. 3 presents a two-dimensional schematic of the concentrator illustrating the dimensions presented on Table 1.

3. RAY-TRACING ANALYSIS METHOD

A program to perform the ray-tracing analysis was created using MatLab and the analysis was performed on LIFE’s two-stage solar concentrator. The following assumptions were made regarding the 2-stage solar concentrator:

1. Every reflection is specular in nature, i.e., the mirror surfaces are reflecting light rays perfectly;
2. Mirrors have perfectly parabolic surfaces;
3. The mount of the 2-stage concentrator is completely rigid, i.e., there is no accounting of possible wind-induced vibration.

The ray-tracing program simulates light as a field of discrete rays, 35,000 were used in the simulations. For every reflection of every ray, three vectors were defined (Fig. 4): the incident ray \( \vec{i} \), the vector normal to the surface of the mirror at the point where the incident ray hits it \( \vec{n} \), and the reflected ray \( \vec{R} \). The path of each ray was calculated using the basic laws of specular reflection [10], mainly:

- The incident ray, normal of the reflective surface, and the reflected ray are all located in the same plane. This was numerically accounted for by making sure the determinant of these three coplanar vectors was equal to zero:

\[
\begin{vmatrix}
I_x & I_y & I_z \\
n_x & n_y & n_z \\
R_x & R_y & R_z
\end{vmatrix} = 0
\]  

(1)
The angle of the reflected ray to the normal of the reflective surface is the same as the angle of the incident ray to the normal. Using the scalar products, the following expression is found:

\[ \theta_I = \theta_R \]  

(2)

\[ \arccos \left( \frac{\vec{I} \cdot \vec{n}}{\|\vec{I}\| \|\vec{n}\|} \right) = \arccos \left( \frac{\vec{R} \cdot \vec{n}}{\|\vec{R}\| \|\vec{n}\|} \right) \]  

(3)

To complete this system of equations, since the three components of \( \vec{R} \) are unknown, the third equation comes from the fact that all three vectors are defined to be unitary:

\[ \|\vec{R}\| = R_x^2 + R_y^2 + R_z^2 = 1 \]  

(4)

Fig. 2. The 2-stage solar concentrator reflects and focus incoming sun rays in a series of reflection: a) Reflection on the primary parabolic mirror focuses the rays on a horizontal focal line, b) Reflection on the secondary parabolic mirror focuses the rays toward the receiver, and c) The concentrated solar energy is used, or transformed, or stored, at or near the receiver plane.

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\[ \|\vec{R}\| = R_x^2 + R_y^2 + R_z^2 = 1 \]  

(4)

Table 1. Solar concentrator dimensions.

<table>
<thead>
<tr>
<th>Mirror</th>
<th>Dimensions (H × W)</th>
<th>Focal Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Mirror</td>
<td>192” × 192”</td>
<td>360”</td>
</tr>
<tr>
<td>Secondary Mirror</td>
<td>96” × 192”</td>
<td>158.8125”</td>
</tr>
<tr>
<td>Inclination of the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Mirror Relative to the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Mirror</td>
<td>70°</td>
<td></td>
</tr>
<tr>
<td>Distance Between both</td>
<td>235”</td>
<td></td>
</tr>
<tr>
<td>Mirrors</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 3. Schematic of the 2-stage solar concentrator accompanied by the principal dimensions.

Fig. 4. Specular reflection: incident (\(\vec{I}\)), normal (\(\vec{n}\)), and the reflected (\(\vec{R}\)) rays.
enabling us to properly solve Eq. (1), (3) and (4) simultaneously. Table 1 presents the various dimensions of the two-stage concentrator relevant to the ray-tracing analysis.

So, sunlight incident on the primary mirror was simulated as a field of parallel rays. As the rays hit the reflective surfaces, they are redirected and concentrated in the focal plane. Fig. 5 shows all the rays simulated and their reflection obtained with the ray-tracing program in the case where the solar concentrator is perfectly aligned with the Sun.

4. RAY-TRACING ANALYSIS METHOD

It was decided that for the purpose of melting metals, if the focal area was smaller than a square of 3 inches (7.62 cm) on the side, enough energy would be concentrated on a small enough area that it would be possible to achieve a desired spot temperature. That decision was made following the observations made after a series of preliminary tests. Having a square target area, 3” × 3”, the solar concentration efficiency of the 2-stage concentrator was defined as:

\[
\eta = \frac{\text{Power incident on the target area}}{\text{Power incident on the primary mirror}} = \frac{P_{\text{target}}}{P_{\text{tot}}} \tag{5}
\]

Having a primary mirror with a total area of 22.3 m² and using the average solar constant of 1 kW/m², the total power incident on the primary mirror (\(P_{\text{tot}}\)) is 22.3 kW. \(P_{\text{target}}\) is calculated with the ray-tracing program for a given 2-stage solar concentrator configuration.

The software also calculated the local solar energy flux \(\Phi\) incident on all parts of the target. To perform that calculation, the surface of the target was discretized into a series of square area, only 0.05” × 0.05” (0.127 cm × 0.127 cm). By calculating the number of rays incident on each of

![Fig. 5. a) Three dimensional and b) side view of the light rays calculated with the ray-tracing program. Dark blue: incident sun rays, red: first reflection rays, yellow: second reflection rays, and light blue: normal vectors on the secondary mirror surface.](image)

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these small square areas, hence knowing the incident energy on these areas, a local solar energy flux $\Phi$, in MW/m², was calculated for every portion of the $3'' \times 3''$ target area.

4.1 Effect of the Focal Length of the Secondary Mirror and the Target Position

In LIFE current design, the focal length of the secondary mirror is 158.8125". Fig. 6a) presents the maximum calculated local solar energy flux on the target as a function of the location of this target in the 2-stage concentrator; as can be seen from Fig. 3, the position of the target $d$ is measured starting at the origin of the system of coordinates used. In that case, the maximum solar energy flux is 1680 MW/m² when the target is position 158.25 cm from the origin. This represents concentration of the sun rays by a factor of $1.68 \times 10^6$. Fig. 6b) shows where the concentrated light rays fall on the $3'' \times 3''$ square target area when it is located 158.25 cm from the origin. It can be observed that the sun rays are focused very tightly in the horizontal direction and very lightly spread over 0.5" in the vertical direction, forming an elongated focal area on the target.

The effect of changing the focal length of the secondary mirror on the maximum local solar energy flux is presented on Fig. 7. The secondary mirror’s focal length providing the highest maximum point-like solar energy flux is 157.9" with $1.2 \times 10^4$ MW/m²; a concentration factor of $1.2 \times 10^7$. Thus, the optimal focal length of the secondary mirror is 157.9", 1" shorter than the focal length initially used by LIFE in their 2-stage solar concentrator prototypes. From Fig. 7, it is also observed that $\Phi$ is strongly dependent on the secondary mirror’s focal length; a change in this focal length of less than an inch results in a decrease in $\Phi$ by a factor of 6.

It can also be observed on Fig. 7 that, apart from the optimal focal length of 157.9", there is two local maxima of $\Phi$ for every focal length studied. Fig. 8 illustrates this observation. On Fig. 8a), for a secondary mirror’s focal length of 157.5", the maximum local solar energy flux is observed for a target displacement of 161.8 cm; there is however a second local maxima for a shorter target displacement of 160.8 cm. The contrary is observed on Fig. 8c) when the
secondary mirror’s focal length of 158.5 cm: the maximum local solar energy flux is observed for a target displacement of 159 cm; a second local maxima is observed for a larger target displacement of 160.5 cm. With the optimal focal length of 157.9 cm, these two maxima coincide, resulting in the maximum value for Φ, as can be seen on Fig. 8b). The occurrence of these two separate maxima is observed when the focal lengths of the two mirrors used in the 2-stage solar concentrator are not properly matched. In that case, the vertical component of the light rays, coming from the primary mirror, will focus at a point that is different than the point where the horizontal component of the rays, coming from the secondary mirror, will focus.

This phenomenon can also be observed visually by looking at the shape of the focal area for target displacement sorter, equal, and longer than the optimal displacement. Fig. 9 presents the shape of the focal area in such a case for a secondary mirror’s focal length of 158.5 cm. For a target displacement from the origin of 159 cm (Fig. 9a), which correspond to one of the maxima, the rays are really well focused horizontally, leaving a vertically shaped focal area. On the other hand, for a target displacement of 160.5 cm (Fig. 9c), which correspond to the other, smaller maxima, the rays are really well focused vertically, leaving a horizontally shaped focal area. In between the maxima (Fig. 9b), the focal area is more rounded since none of the rays are actually focused exactly in any direction.

Figure 10 shows the calculated concentration efficiency (η) as a function of the target displacement from the origin for the LIFE’s initial secondary mirror’s focal length of 158.8125 cm. This figure clearly demonstrate that the exact placement of the target area does not play a crucial role when it comes to ensure that all the incident energy on the concentrator will
ultimately land on the specified 3” × 3” target area. As can be seen from Fig. 10, the concentration efficiency \( \eta \) is equal to 1 for a target displacement from the origin \( d \) that varies from 153 to 164 cm. The nature of this result was not affected by the change in secondary mirror’s focal length in the range presented in this study.

4.2 Effect of the Alignment of the 2-Stage Solar Concentrator with the Sun

Because the Sun is constantly moving westward in the sky, it is important to know how a misalignment between the 2-stage solar concentrator’s primary mirror and the sun will affect the concentration efficiency of the system; and as a result, how accurate a tracking system installed on the concentrator will have to be.
Figure 11 presents the resulting variation in the concentration efficiency as a function of vertical or horizontal misalignment. These concentrations were calculated using LIFE’s initial secondary mirror’s focal length of 158.8125 in. The effect of small changes in this focal length when it comes to misalignments is negligible. Fig. 11a) shows that only a very small misalignment of $\pm 0.2$ in is permitted in the vertical direction in order to maintain a concentration efficiency of close to a 100%; this becomes $\pm 0.3$ in the horizontal direction (Fig. 11b). Notice that the graph on Fig. 11b) is symmetrical as it should be.

5. CONCLUSIONS

A ray-tracing analysis was conducted on a 2-stage solar concentrator made of two parabolic mirrors created by Lunenburg Industrial Foundry & Engineering (LIFE). A ray-tracing
A program was created using MatLab and the effects of the secondary mirror’s focal length, the distance secondary mirror and the target, and the misalignment with the sun were studied. It was found that the focal length of the secondary mirror played a role only when it was a question of the maximum local solar energy flux that can be achieved on the target. For the

![Concentration efficiency as a function of target displacement](image1)

Fig. 10. Concentration efficiency $\eta$ as a function of the target displacement from the origin $d$ for a secondary mirror’s focal length of 158.8125 cm.

![Effect of misalignment on concentration efficiency](image2)

Fig. 11. Effect of a misalignment of the primary mirror with the sun on the concentration efficiency $\eta$ of the collector: a) Vertical alignment and b) Horizontal alignment.

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optimal focal length of 157.9°, a maximum $\Phi = 1.2 \times 10^4$ MW/m² was achieved compared to $\Phi = 1680$ MW/m² for the initial LIFE’s focal length of 158.8125°. The choice of the secondary mirror’s focal length also influenced the shape of the focal area on the target.

It was found that all the energy from the sun was concentrated on the target, and that is independent of the secondary mirror’s focal length (within the range studied), as long as the target position is within an 11 cm zone.

Finally, it was discovered that such a system is very sensitive to misalignments with respect to the sun. Small misalignments in the order of $\pm 0.2°$ would bring the concentration efficiency of LIFE’s 2-stage concentrator to essentially zero. A well designed guidance control system will have to be installed on the concentrator to maximize the energy output throughout entire sunny periods. This guidance system requiring movement around only one axis should be inexpensive and simple to build, using an electric motor and a set of reduction gears to adjust to the known speed at which the sun is travelling in the sky.

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